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Branching Semigroups with a Compatible Set-Valued Operator

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ABSTRACT

This paper introduces a branching semigroup, defined as a semigroup equipped with an extensive set-valued unary operator that is compatible with semigroup multiplication. The construction separates deterministic composition from nondeterministic branching while retaining enough algebraic structure to support ideals, homomorphisms, quotients, and graph-based analysis. We prove that ordinary semigroups embed as singleton-branching structures, intersections of branch ideals are branch ideals, strong homomorphic images inherit the branching structure, and kernels produce quotient branching semigroups satisfying a first isomorphism theorem. For finite structures, ascending and descending chains of branch ideals stabilize. We associate a directed branch graph to every branching semigroup and obtain a canonical decomposition into strongly connected components; sink components are branch-closed, although components need not be subsemigroups without an additional closure assumption. Two fully verified examples are given: a coset-based structure on the cyclic group of order four and a truncated-addition monoid with an explicit chain of branch ideals. Potential uses in nondeterministic composition, routing, and decision systems are presented as modeling interpretations rather than empirical claims.

1. Introduction

In a conventional automotive or small industrial alternator, the stator carries the output winding while the rotor carries the field winding. A voltage regulator controls the field current, and two stationary brushes transfer that current through two rotating slip rings. This arrangement is compact and controllable, but it creates a continuously sliding electrical contact whose condition directly affects excitation stability and therefore alternator output. Standard terminology for brushes, brush holders, commutators, and slip rings is defined by IEC 60276, while IEC 60136 addresses brush and brush-holder dimensions, marking, and manufacturing tests [1,2].

The original manuscript correctly recognized that slip-ring damage is multi-physical, but several claims were stated as universal numerical limits even though brush grade, ring alloy, geometry, ventilation, regulator strategy, duty cycle, and manufacturer design strongly influence acceptable values. This revised paper therefore emphasizes mechanisms, trends, and diagnostic logic. Numerical values are used only when tied to a cited experiment or when clearly identified as an equipment-specific limit rather than a general rule.

The scope is limited primarily to copper or copper-alloy rings operating with carbon-graphite or metal-graphite brushes in automotive and compact industrial alternators. Large synchronous-generator collector systems may use different ring materials, brush grades, current densities, ventilation arrangements, and maintenance criteria; their limits should not be transferred directly to automotive alternators.

2. Contact Physics and Functional Requirements

2.1 Real contact area and constriction resistance

A nominally flat brush and ring touch only at microscopic asperities. The sum of these small junctions is the real contact area, which is much smaller than the apparent brush footprint. Current therefore converges into a limited number of conductive paths, creating constriction resistance. Holm's contact theory provides the classical framework for this behavior [4]. For a simplified circular contact spot of radius a between two conductors of resistivity rho1 and rho2, the constriction component may be represented approximately by:

Rc approximately equals (rho1 + rho2) / (4a)

The equation is a local idealization, not a complete model of a working brush. A practical interface also contains bulk brush resistance, lead and terminal resistance, surface-film resistance, multiple time-varying contact spots, and intermittent micro-separation. The total contact voltage is therefore better treated as a measured system response than as a fixed material constant.

Vcontact = I x (Rbrush + Rfilm + Rconstriction + Rconnections)

At each conductive microjunction, electrical power is dissipated as I squared R. Friction contributes additional heat. The resulting temperature field is highly nonuniform, so a modest average brush temperature can coexist with much hotter microscopic junctions. This is why trend monitoring and comparison among paired brushes are often more useful than a single surface-temperature reading.

2.2 Sliding speed and dynamic contact

The peripheral speed of a cylindrical slip ring is determined by ring diameter and rotational speed:

v = pi x D x n

where v is peripheral speed, D is ring diameter, and n is rotational speed in revolutions per second. Increasing speed changes frictional heating, air drag, brush vibration, and the time available for individual asperities to carry current. Brush-holder clearance, spring characteristics, rotor runout, bearing condition, and surface waviness determine whether the brush follows the ring continuously. Experiments on dynamically excited metal-graphite contacts confirm that rotational speed, current, temperature, and brush dynamics interact rather than acting as independent variables [6].

3. Materials and Surface-Film Chemistry

3.1 Slip-ring materials

Copper is widely used because of its high electrical and thermal conductivity and its ability to form a workable oxide and graphite transfer film. Alloying additions may improve strength, softening resistance, or wear behavior, but they also alter conductivity, oxide chemistry, machinability, and compatibility with the selected brush grade. Copper-silver and copper-chromium-zirconium alloys are plausible choices in demanding designs, yet no alloy should be described as universally superior. The ring material must be qualified as a pair with the brush, spring force, speed range, and environment.

3.2 Brush materials

The brush is intentionally more replaceable than the ring and normally contains graphite, carbon, metal powder, binders, and application-specific additives. Metal-graphite grades are common where low voltage drop and relatively high current capability are required. Electrographitic grades offer different friction, filming, and resistivity characteristics and may be preferred in other rotating machines. IEC standards define nomenclature and test frameworks, but the manufacturer must select the grade for the actual machine [1-3].

Material family	Engineering advantages	Important limitations
Copper or copper-alloy ring	High conductivity; good heat spreading; readily machinable; compatible with many metal-graphite brushes.	Can soften, groove, oxidize, or arc-erode; performance depends on alloy and surface condition.
Metal-graphite brush	Low bulk resistivity; suitable for low-voltage excitation; graphite supports solid lubrication.	Metal content changes friction, hardness, oxidation, and ring wear; grade substitution is risky.
Electrographitic brush	Good thermal stability and filming in many rotating-machine duties.	Usually higher contact drop than heavily metalized grades; may not suit every automotive design.

3.3 Tribofilm or patina

A mature contact does not operate as clean bulk copper sliding directly against an unchanged brush face. The interface develops a tribofilm containing copper oxides, transferred graphite, wear debris, and adsorbed species from the atmosphere. Research on graphite-copper contacts shows that atmosphere, current direction, surface morphology, and film condition influence friction, voltage drop, and wear [7-9]. The film can protect the ring and stabilize conduction, but it is dynamic: excessive removal raises mechanical wear, whereas excessive insulating contamination or oxide growth can raise contact resistance and promote unstable conduction.

Humidity is therefore not governed by one universal relative-humidity window. Very dry conditions can reduce the beneficial role of adsorbed species and increase brush wear in some systems, while high humidity can change oxidation and contamination behavior. Hu, Chen, and Xia reported a strong humidity effect for a particular electrographite-copper commutator experiment [7]. That result supports humidity control as an engineering consideration, but it does not justify applying one numerical limit to every alternator.

4. Wear and Damage Mechanisms

4.1 Abrasive wear

Two-body abrasion occurs when hard asperities or hard constituents cut the opposing surface. Three-body abrasion occurs when external dust or internally generated debris rolls or slides between brush and ring. Parallel grooves aligned with the direction of motion are common evidence. Turel and co-workers observed abrasive grooves in metal-graphite brush tests, and Senouci and co-workers linked graphite-copper wear to surface morphology and contact conditions [6,8]. Abrasion is accelerated by contamination, an unsuitable brush grade, excessive roughness, poor cleaning practice, or excessive normal force.

4.2 Adhesive wear and material transfer

If the protective surface film is disrupted and local temperature and pressure are high, junctions may form between contacting asperities. Their subsequent shear transfers material or tears small fragments from the ring. Adhesive wear can appear as smearing, pickup, scoring, or local roughening rather than uniform polishing. The mechanism is especially likely when lubrication by the tribofilm is weak, when the ring has softened thermally, or when contamination has removed or chemically altered the film.

4.3 Electrical discharge erosion

A brush that loses contact while field current is flowing can produce a transient voltage across the opening gap. If breakdown occurs, a micro-arc removes or resolidifies a small volume of material and damages the tribofilm. Repeated events produce pitting, dark marks, local craters, and increased electrical noise. The cause is not simply 'high voltage'; it is usually a chain involving runout, bearing vibration, brush sticking, inadequate spring force, poor holder geometry, contamination, or an abnormal excitation transient. Manufacturer guidance likewise treats sparking as evidence of excessive voltage difference associated with interrupted contact rather than as a standalone failure mode [15].

4.4 Polarity-dependent asymmetry

Opposite rings carry opposite current directions, so their films and wear rates may differ. However, the statement that one polarity always experiences a specific type of erosion is too absolute. Laboratory studies have reported anode-cathode differences, but the direction and magnitude depend on material pair, atmosphere, current density, film chemistry, and test geometry [8,9]. Maintenance decisions should be based on measured asymmetry between the two rings, not on an assumed universal polarity rule.

4.5 Thermal and chemical degradation

Heat changes brush resistivity, binder behavior, spring properties, ring hardness, oxidation rate, and contaminant chemistry. A hot interface may show lower contact resistance while simultaneously suffering greater wear, so low resistance alone does not prove good condition. Turel and co-workers demonstrated that the relationships among temperature, current, speed, resistance, and wear were condition-dependent [6].

Oil, coolant, salt, sulfur-bearing gases, and cleaning residues can create insulating glaze, corrosive products, sticky debris, or altered films. Chemical composition should therefore be investigated when surface appearance cannot be explained by mechanics alone.

5. Operating Variables and Their Interactions

5.1 Current density and electrical load

Higher field current increases electrical heating and may change film formation. It also changes the number and size of active conduction spots. Tests on copper-impregnated metallized carbon against copper alloy showed that electrical current can alter both friction and wear [14]. The response is nonlinear: at some conditions a minimum electrical load may help establish stable conduction, while excessive load or poor contact concentrates heating and accelerates damage. Current density should therefore be compared with the brush manufacturer's allowable range for the actual grade and duty cycle.

5.2 Spring force and holder condition

Too little normal force allows contact loss and arcing; too much force increases friction and mechanical wear. The optimum is not a universal pressure band. In one copper-graphite laboratory study, minimum wear occurred within a particular 30 to 50 kPa range, but the result was specific to that material pair and test configuration [11]. In service, the correct force is the equipment or brush supplier specification, verified with a calibrated method. Equally important are free brush movement, correct holder clearance, aligned holders, intact flexible leads, and springs that deliver similar force at the installed brush lengths.

5.3 Surface geometry and roughness

Runout, taper, waviness, eccentricity, and local flats cause periodic modulation of contact force. Surface roughness influences seating, film retention, friction, and voltage drop, but a single universal Ra range cannot be applied across all ring materials and brush grades. The acceptance criterion should come from the alternator or brush manufacturer and should specify measurement method, filter, sampling length, and whether the value applies to a newly machined or run-in surface.

5.4 Environment and duty cycle

Under-hood alternators experience large temperature cycles, dust, road salt, water splash, oil mist, and rapid speed changes. Industrial alternators may instead face long steady-state runs, controlled ventilation, or process gases. Service life must be interpreted against duty cycle.

A ring that survives high speed at moderate current may perform differently at low speed and high current because the thermal balance, film formation, and vibration response have changed. This interaction explains why single-factor wear curves often fail to predict field life.

6. Failure Modes and Diagnostic Evidence

Diagnosis should combine visual evidence with geometry, electrical measurements, brush condition, and operating history. Surface color alone is insufficient because a dark film may be normal, overheated, contaminated, or electrically damaged. The following matrix provides a structured starting point.

Observed condition	Likely mechanisms and confirming evidence	Recommended checks and actions

Uniform polished track; stable brush wear	Normal running-in and stable tribofilm. Similar appearance on both rings; no abnormal ripple or heat.	Record as baseline. Continue trend monitoring and verify brush length and spring force.
Parallel grooves or circumferential scoring	Abrasive particles, excessive roughness, hard brush constituents, or excessive force. Debris may be embedded in brush face.	Inspect filtration and seals; examine brush face; measure roughness and runout; correct cause before resurfacing.
Localized pits, craters, or burned spots	Intermittent contact and electrical discharge erosion. Often accompanied by electrical noise, brush bounce, or periodic voltage disturbance.	Check runout, bearings, holder movement, spring force, lead connections, and excitation transients.
Thick glaze, sticky black deposit, or uneven film	Oil or fluid contamination, overheated residue, excessive oxide, or mixed debris. May cause high or unstable contact voltage.	Identify contaminant source; clean using approved procedure; replace contaminated brushes if required.

Unequal wear between rings or brushes	Polarity-related film asymmetry, misalignment, unequal force, lead resistance, holder friction, or unequal cooling.	Measure both paths separately; compare force, temperature, voltage, geometry, and surface chemistry.
Periodic electrical dropout at shaft frequency	Ring eccentricity, local flat, bearing motion, or holder dynamics.	Correlate oscilloscope or current waveform with rotational speed; measure total indicated runout.

6.1 Electrical measurements

Useful measurements include field-current stability, voltage across each brush path, resistance of flexible leads and terminals, alternator output ripple, and correlation of disturbances with shaft speed. Absolute contact-voltage limits must come from the equipment design because voltage includes brush grade, current, temperature, number of contacts, wiring, and measurement location. A rising or increasingly variable voltage trend under comparable conditions is generally more meaningful than an isolated reading.

6.2 Thermal measurements

Infrared inspection can identify unequal heating, but emissivity, viewing angle, airflow, and access can distort readings. The most reliable use is comparative: measure both brushes and nearby hardware under the same load and repeat the measurement at consistent locations. A hot brush should trigger checks of current sharing, contact stability, spring force, brush freedom, lead resistance, and contamination rather than an automatic conclusion that the brush grade is defective.

6.3 Geometry and surface metrology

During overhaul, measure ring diameter, radial runout, axial position, taper, groove depth, and roughness with traceable instruments. Optical profilometry and microscopy can distinguish ploughing, smearing, transfer, and cratered recast material. Scanning electron microscopy and energy-dispersive X-ray spectroscopy are useful for laboratory root-cause work, especially where salt, sulfur, silicon, iron-rich debris, or unusual oxide chemistry is suspected. These methods support diagnosis; they do not replace measurements of the assembled rotor, holder, and spring system.

7. Maintenance and Life-Extension Framework

7.1 Establish an equipment-specific baseline

Immediately after commissioning or a controlled overhaul, record brush grade, ring material, ring diameter, surface finish, radial runout, brush length, spring force, holder clearance, field current, contact voltage, temperature, and representative waveforms. A baseline converts later inspections from subjective appearance checks into trend analysis. IEC 60773 provides standardized laboratory methods for brush operational characteristics, but in-service acceptance limits still require machine-specific engineering judgment [3].

7.2 Control assembly variables

- Use the specified brush grade; do not mix grades or substitute by appearance.
- Replace brushes as a matched set when required by the manufacturer and seat them using an approved method.

- Verify free movement in each holder and remove burrs, glaze, or contamination without enlarging the holder.
- Measure spring force at a defined brush length and replace springs that are weak, damaged, overheated, or inconsistent.
- Check flexible leads and terminals for looseness, fatigue, corrosion, or localized heating.
- Correct bearing condition, shaft runout, and rotor imbalance before treating the ring surface.

7.3 Control contamination and cooling

Seals, baffles, drainage, and ventilation should keep oil, coolant, road salt, and abrasive dust away from the contact. Cleaning agents must be compatible with insulation, brush binders, and plastics and must leave no residue.

Compressed air can drive conductive dust deeper into the alternator and should be used only under an approved procedure with appropriate collection. Any repeated glaze or corrosion should be traced to its source rather than repeatedly polished away.

7.4 Resurfacing and replacement decisions

Resurfacing is justified only when sufficient material remains and the corrected geometry, finish, insulation clearance, and concentricity can meet the manufacturer's requirements. Polishing a ring while leaving runout, bearing motion, contamination, or an incorrect brush grade unchanged usually produces a short-lived repair.

Replacement is preferable when grooves are deep, the ring is cracked or loose, insulation is damaged, minimum diameter would be exceeded, or repeated electrical erosion indicates a system-level defect.

7.5 Condition-based inspection plan

Measurement or observation	Purpose	Trend or trigger logic
Brush length and wear rate	Estimate remaining life and detect unequal loading.	Compare equal service intervals and paired brushes; investigate acceleration or asymmetry.
Spring force and brush freedom	Maintain stable mechanical contact.	Compare with supplier specification; investigate spread within the set.
Field current and contact voltage	Detect unstable conduction or resistive joints.	Trend under similar load and temperature; investigate drift or increased variation.
Temperature comparison	Identify current concentration, friction, or poor connections.	Compare paired components under the same operating condition.
Output or field-current waveform	Detect periodic contact loss or excitation disturbance.	Correlate disturbances with rotational frequency and operating events.
Runout and surface profile	Quantify geometry-related brush motion and wear.	Compare with machine-specific limit and previous overhaul data.
Contamination and film appearance	Identify environmental or chemical causes.	Photograph consistently; confirm suspected contaminants by analysis when necessary.

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7.6 Emerging technologies

Laser surface texturing, hard composite coatings, nanoparticle-reinforced materials, and alternative brush compositions are active research areas. They may improve debris retention, hardness, friction, or electrical stability in a controlled test. However, a coating or texture also changes current constriction, film growth, thermal conduction, roughness evolution, and compatibility with the brush. These approaches should therefore be described as promising rather than established universal upgrades. Validation should include endurance, thermal cycling, contamination, overspeed, vibration, and electrical-transient tests on the actual material pair.

8. Conclusion

Alternator slip-ring wear is governed by the coupled behavior of the brush, ring, tribofilm, spring-holder system, excitation circuit, rotor dynamics, and environment. Abrasion, adhesion, electrical discharge erosion, thermal degradation, contamination, and polarity-dependent film asymmetry can occur together and can produce similar visual symptoms. Consequently, reliable diagnosis requires correlation of surface evidence with brush wear, spring force, holder freedom, contact voltage, temperature, waveform stability, runout, and operating history.

The most important revision to the original analysis is the rejection of unsupported universal limits. Humidity, contact pressure, voltage drop, surface roughness, and temperature must be evaluated for the selected brush grade, ring material, geometry, and duty cycle. Standardized terminology and test methods provide a sound framework [1-3], while published experiments demonstrate the strong interactions among current, speed, temperature, pressure, atmosphere, and dynamics [6-14].

In practice, the longest service life is achieved by correct material pairing, stable spring force, accurate alignment and runout control, clean ventilation, sound electrical connections, and disciplined trend monitoring. Advanced coatings or surface textures may add value, but they should supplement rather than replace these basic controls. A condition-based program built on an equipment-specific baseline is more defensible than maintenance based on fixed generic thresholds or surface appearance alone.

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