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An Odd–Even Cost Reduction Heuristic with MODI Refinement for Solving Transportation Problems

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ABSTRACT

The transportation problem is a significant optimization model in operations research, logistics, and supply-chain management. Its main objective is to determine a minimumcost shipping plan from multiple origins to multiple destinations while satisfying all supply and demand constraints. Existing methods such as the North-West Corner Method, Least Cost Method, and Vogel's Approximation Method are commonly used to obtain an Initial Basic Feasible Solution. This paper revisits an odd–even cost reduction heuristic proposed for transportation problems and evaluates its effectiveness as an initial-solution method. The analysis shows that the odd–even heuristic can generate better initial solutions than the Least Cost Method in selected examples. However, the method alone does not guarantee optimality. Therefore, this study proposes a corrected hybrid approach in which the oddeven heuristic is first used to obtain an Initial Basic Feasible Solution, followed by the Modified Distribution Method to test and improve optimality. Numerical results show that the proposed hybrid approach reduces the total transportation cost from 779 to 743 in the f irst example and from 138 to 128 in the second example. The study concludes that the odd–even method should be considered an initial-solution technique rather than a complete optimization method.

1 Introduction

The transportation problem is one of the classical problems in operations research and linear programming. It deals with the distribution of a homogeneous product from several sources to several destinations at minimum total transportation cost. Each source has a fixed supply, and each destination has a fixed demand. The objective is to determine how much quantity should be transported from each source to each destination so that all supply and demand conditions are satisfied at the lowest possible cost.

Transportation models are widely used in logistics, production planning, warehousing, supply-chain management, agricultural distribution, industrial transportation, and resource allocation. Because transportation cost often forms a large part of total operational cost, even a small improvement in the transportation plan can produce significant economic benefits.

In transportation-problem solution procedures, the first step is usually to find an Initial Basic Feasible Solution. Several classical methods are available for this purpose, including the North-West Corner Method, Least Cost Method, and Vogel's Approximation Method. Among these, the Least Cost Method uses transportation costs directly and often gives a better starting solution than the North-West Corner Method. However, an Initial Basic Feasible Solution is not necessarily optimal. Therefore, an optimality test such as the Modified Distribution Method or stepping-stone method must be applied after obtaining the initial solution.

Gupta and Henry proposed an odd–even cost-based method for obtaining an Initial Basic Feasible Solution to a transportation problem. Their method compared favorably with the Least Cost Method in selected numerical examples. However, the original study did not apply a formal optimality test. As a result, the obtained solutions were better than the Least Cost Method solutions but were not necessarily optimal.

Therefore, the present paper revises and extends the odd–even cost reduction approach by combining it with the Modified Distribution Method. The purpose is to clearly distinguish between an initial feasible solution and an optimal solution.

2 Mathematical Formulation of the Transportation Problem

Suppose there are m sources and n destinations.

Let

$$a_i = \text{supply available at source } i,$$

$$b_j = \text{demand required at destination } j,$$

$$c_{ij} = \text{unit transportation cost from source } i \text{ to destination } j,$$

and

$$x_{ij} = \text{quantity transported from source } i \text{ to destination } j.$$

The mathematical model of the transportation problem is given as follows:

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \ldots, m,$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \ldots, n,$$

$$x_{ij} \geq 0.$$

The transportation problem is balanced if

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j.$$

If total supply is not equal to total demand, a dummy source or dummy destination is added to balance the problem.

3 Review of Existing Initial-Solution Methods

3.1 North-West Corner Method

The North-West Corner Method begins allocation from the top-left corner of the transportation table. It is simple and easy to apply, but it does not consider transportation costs during allocation. Therefore, it may produce a poor Initial Basic Feasible Solution.

3.2 Least Cost Method

The Least Cost Method begins allocation from the cell with the lowest transportation cost. It usually gives a better solution than the North-West Corner Method because it considers cost values. However, it is still only an initial-solution method and does not guarantee optimality.

3.3 Vogel's Approximation Method

Vogel's Approximation Method uses penalty costs based on the difference between the two smallest costs in each row and column. It usually gives a better Initial Basic Feasible Solution than the North-West Corner Method and Least Cost Method. However, it also requires an optimality test before the solution can be accepted as optimal.

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3.4 Modified Distribution Method

The Modified Distribution Method, commonly called the MODI method, is used to test whether a transportation solution is optimal. If the solution is not optimal, MODI identifies an entering cell and improves the allocation until no further cost reduction is possible.

4 Odd–Even Cost Reduction Heuristic

The odd–even cost reduction heuristic is based on the separation of odd and even costs in the transportation matrix. The main idea is to subtract the least even cost from all even costs and the least odd cost from all odd costs. This produces at least one zero-cost cell, which is then used for allocation.

The steps are as follows:

1. Balance the transportation problem if it is not already balanced.
2. Check whether any zero-cost cell exists.
3. If a zero-cost cell exists, allocate the minimum of corresponding supply and demand to that cell.
4. If no zero-cost cell exists, identify the least even cost and the least odd cost.
5. Subtract the least even cost from all even costs.
6. Subtract the least odd cost from all odd costs.
7. Allocate to the zero-cost cell according to available supply and demand.
8. Eliminate the exhausted row or column.
9. Repeat the process until all supply and demand requirements are satisfied.
10. Compute the transportation cost using the original cost matrix.

This heuristic can produce a good Initial Basic Feasible Solution. However, it should not be treated as an optimality method because it does not use reduced costs or optimality conditions.

5 Proposed Hybrid Method

The revised method proposed in this paper combines the odd–even cost reduction heuristic with the MODI method.

The proposed method has two stages.

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5.1 Stage 1: Initial Solution Generation

Use the odd–even cost reduction heuristic to obtain an Initial Basic Feasible Solution.

5.2 Stage 2: Optimality Testing and Improvement

Apply the MODI method to test whether the solution is optimal.

For every basic cell (i, j), determine row and column potentials u_i and v_j such that

$$u_i + v_j = c_{ij}.$$

For every non-basic cell, calculate

$$\Delta_{ij} = c_{ij} - (u_i + v_j).$$

For a minimization problem:

- If all $\Delta_{ij} \geq 0$, the solution is optimal.
- If any $\Delta_{ij} < 0$, the solution is not optimal and can be improved.

When a negative reduced cost exists, the cell with the most negative value is selected as the entering cell. A closed loop is formed, allocations are adjusted, and the process is repeated until all reduced costs are non-negative.

6 Numerical Analysis

6.1 Example 1

Consider the following transportation problem:

Table 1: Transportation cost matrix for Example 1

	D_1	D_2	D_3	D_4	Supply
O_1	19	30	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
Demand	5	8	7	14	34

The problem is balanced because total supply and total demand are both 34.

Using the odd–even cost reduction heuristic, the following allocation is obtained:

$$x_{11} = 5, \quad x_{14} = 2, \quad x_{32} = 8, \quad x_{23} = 7, \quad x_{24} = 2, \quad x_{34} = 10.$$

The transportation cost is

$$Z = 5(19) + 2(10) + 8(8) + 7(40) + 2(60) + 10(20),$$

$$Z = 95 + 20 + 64 + 280 + 120 + 200,$$

$$Z = 779.$$

Thus, the odd–even method gives an Initial Basic Feasible Solution with total cost

$$Z = 779.$$

However, this solution must be tested for optimality.
Using the MODI method, the reduced cost for cell $(2, 2)$ is

$$\Delta_{22} = -18.$$

Since $\Delta_{22} < 0$, the solution is not optimal.
After adjustment through the MODI method, the improved allocation is

$$x_{11} = 5, \quad x_{14} = 2, \quad x_{22} = 2, \quad x_{23} = 7, \quad x_{32} = 6, \quad x_{34} = 12.$$

The improved transportation cost is

$$Z = 5(19) + 2(10) + 2(30) + 7(40) + 6(8) + 12(20),$$

$$Z = 95 + 20 + 60 + 280 + 48 + 240,$$

$$Z = 743.$$

Therefore, the optimal cost for Example 1 is

$$Z = 743.$$

6.2 Example 2

Consider the following transportation problem:

The problem is balanced because total supply and total demand are both 31.

Using the odd–even cost reduction heuristic, the following allocation is obtained:

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Table 2: Transportation cost matrix for Example 2

	D_1	D_2	D_3	Supply
O_1	6	8	4	14
O_2	4	9	3	12
O_3	1	2	6	5
Demand	6	10	15	31

$$x_{31} = 5, \quad x_{21} = 1, \quad x_{12} = 10, \quad x_{13} = 4, \quad x_{23} = 11.$$

The total transportation cost is

$$Z = 5(1) + 1(4) + 10(8) + 4(4) + 11(3),$$

$$Z = 5 + 4 + 80 + 16 + 33,$$

$$Z = 138.$$

Thus, the odd–even method gives an Initial Basic Feasible Solution with total cost

$$Z = 138.$$

However, the MODI optimality test shows that the reduced cost for cell $(3, 2)$ is

$$\Delta_{32} = -2.$$

Since $\Delta_{32} < 0$, the solution is not optimal.
After applying the MODI adjustment, the improved allocation is

$$x_{12} = 5, \quad x_{13} = 9, \quad x_{21} = 6, \quad x_{23} = 6, \quad x_{32} = 5.$$

The improved cost is

$$Z = 5(8) + 9(4) + 6(4) + 6(3) + 5(2),$$

$$Z = 40 + 36 + 24 + 18 + 10,$$

$$Z = 128.$$

Therefore, the optimal cost for Example 2 is

$$Z = 128.$$

7 Results and Discussion

The comparison of the Least Cost Method, odd–even heuristic, and proposed hybrid method is shown in Table 3.

Table 3: Comparison of transportation costs

Example	Least Cost Method	Odd–Even Heuristic	Proposed Hybrid Method
Example 1	814	779	**743**
Example 2	139	138	**128**

The results show that the odd–even heuristic improves the Least Cost Method in both examples. In Example 1, the cost decreases from 814 to 779. In Example 2, the cost decreases from 139 to 138.

However, after applying the MODI method, further cost reductions are achieved. In Example 1, the cost decreases from 779 to 743. In Example 2, the cost decreases from 138 to 128.

This confirms that the odd–even heuristic is useful as an initial-solution method, but it is not sufficient as a complete optimization method. The method gives a feasible starting solution, but optimality can only be confirmed after applying an optimality test.

Therefore, the proposed hybrid method is more reliable because it combines the simplicity of the odd–even heuristic with the mathematical accuracy of the MODI method.

8 Conclusion

This paper revised and extended the odd–even cost reduction approach for solving transportation problems. The original odd–even method is useful for generating an Initial Basic Feasible Solution and may produce better results than the Least Cost Method. However, the method does not guarantee optimality.

The numerical analysis shows that the odd–even method gives transportation costs of 779 and 138 in the two examples. After applying the MODI method, these costs are reduced to 743 and 128 respectively. Therefore, the corrected approach shows that the odd–even method should be used only as an initial-solution procedure, followed by MODI or another optimality-testing method.

The main contribution of this revised study is the development of a more accurate hybrid framework for transportation problems. This framework clearly separates the initial feasibility stage from the optimality-verification stage.

Future studies may test the proposed hybrid approach on larger transportation problems, unbalanced transportation problems, degenerate cases, and real-world logistics data. Further comparison with Vogel's Approximation Method, improved VAM, stepping-stone method, and other modern heuristics is also recommended.

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