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# Takagi–Sugeno Fuzzy Systems with Input and Output Constraints: A Review of Robust, Optimal, and Predictive Control Methods

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**ABSTRACT**

This paper reviews advanced control techniques for Takagi–Sugeno (T-S) fuzzy systems that are subject to input and output constraints. These constraints are essential in real-time applications, such as aerospace, automotive, robotics, and industrial systems, where physical limitations dictate the system's performance. The paper discusses key methodologies such as Model Predictive Control (MPC), anti-windup compensators, and Linear Matrix Inequality (LMI)-based controls, which ensure robust, optimal, and predictive performance while adhering to these constraints. The review also addresses recent advancements in adaptive control strategies and highlights opportunities for integrating data-driven methods to improve scalability and robustness in high-uncertainty environments. We provide insights into how these control strategies contribute to stability, robustness, and performance optimization in T-S fuzzy systems, and propose future research directions that explore scalable solutions for large-scale systems.

## 1 Introduction

Control systems must account for both input and output constraints to maintain safety and performance, especially in practical, real-world applications. Input constraints arise due to limitations in actuators and control signals, such as saturation (e.g., motor torque, voltage, fuel). Failing to address these constraints can lead to instability and poor performance. For example, in flight control systems, the deflection angle of aircraft surfaces must not exceed certain limits to maintain safe flight. Similarly, in robotics, motors are limited by their maximum torque and speed.

Takagi-Sugeno (T-S) fuzzy systems are commonly used to model and control nonlinear systems by decomposing them into linear approximations based on fuzzy rules. These systems are highly beneficial for handling uncertainties and nonlinearities in complex applications. However, T-S fuzzy systems face significant challenges when subjected to input/output constraints. These constraints require advanced control methods that ensure system stability and optimal performance.

This paper presents a comprehensive review of advanced control techniques for T-S fuzzy systems, focusing on the treatment of input and output constraints. We review robust, optimal, and predictive control methods such as MPC, anti-windup compensators, and LMI-based techniques to guarantee stability and performance in constrained systems. Additionally, we explore recent adaptive control strategies and data-driven methods, addressing research gaps and future directions for integrating these strategies into real-time control systems.

## 2. Mathematical Foundations of Takagi–Sugeno Fuzzy Systems

T-S fuzzy systems represent nonlinear systems by combining multiple linear models using fuzzy membership functions. The general form of the T-S fuzzy model is given by:

Plant Rule i:

If $\Psi_1(t)$ is $Mi_1$ and $\Psi_2(t)$ is $Mi_2$ and … and $\Psi q(t)$ is $Mi_q$,

then:

$$\frac{dy}{dt}x(t) = A_i x(t) + B_u u(t) + B_v v(t) \tag{1}$$

$$y(t) = C_i x(t) \tag{2}$$

Where $x(t)$ is the state vector, $u(t)$ is the control input, $v(t)$ is the external disturbance, and $A_i$, $B_u$, $B_v$, $C_i$ are the system matrices. The fuzzy rules define how these variables interact under different operational conditions, with the output being the weighted sum of the local linear models.

These models can be applied to various systems, from robotic arms to autonomous vehicles. In particular, the flexibility of T-S fuzzy systems in capturing nonlinear dynamics has made them crucial in control applications for systems with significant uncertainty and multi-input, multi-output (MIMO) constraints (e.g., [1], [10], [13]).

## 3. Input Constraints: Actuator Saturation

Input constraints, such as actuator saturation, occur when the control inputs exceed the physical capabilities of the actuators. This saturation leads to loss of control authority, causing performance degradation or even system failure. The saturation function is defined as:

$$u(t) = sat(u_e(t)) \tag{3}$$

where ue(t)u_e(t)ue(t) is the control input before saturation, and the saturation function is given by:

$$\text{sat}(u_e) = \begin{cases} u_e L, \text{ if } u_e < u_e L \\ u_e, \text{ if } u_e L \le u_e \le u_e H \\ u_e, \text{ if } H u_e > u_e H \end{cases}$$

Where $u_e L$ and $u_e H$ are the lower and upper bounds of the control input.

To handle actuator saturation, anti-windup compensators are typically employed. These compensators adjust the control signal when saturation occurs, allowing the system to continue operating effectively despite actuator constraints. Robust control methods such

as H∞ control are also used to minimize the effects of uncertainties and external disturbances, especially in systems subject to input saturation ([5], [9]).

## 4. Output Constraints: Passivity and Stability

Output constraints are essential to ensure the system operates within safe limits. These constraints are critical in fields like aerospace, where a system's output (such as temperature or pressure) must stay within specific bounds to avoid hazardous conditions. Passivity-based control is often used to ensure that the system remains energy-stable and can dissipate external energy inputs.

The passivity condition for T-S fuzzy systems is given by:

$$\int_0^{t_p} y(t)^T S v(t) dt > \gamma \int_0^{t_p} v(t)^T v(t) dt \tag{5}$$

for all $t_p \geq 0$ and $v(t) \neq 0$, whereS is a constant matrix and $\gamma$ is a scalar. This ensures that the system can handle disturbances effectively and remain stable.

Passivity is also related to ensuring that the system does not violate stability constraints under disturbances, as shown in recent works on fuzzy control under passivity constraints ([7], [15]).

## 5. Robust Control Methods

### 5.1 H∞ Control:

H∞ control minimizes the worst-case effect of disturbances and uncertainties, ensuring that the system remains stable even under extreme conditions. It is particularly useful when the system is exposed to significant external disturbances ([12], [15]).

### 5.2 Anti-Windup Compensators:

Anti-windup compensators adjust the controller output to prevent integrator windup when saturation occurs. This ensures that the system remains stable even when actuator limitations are reached, preventing degradation of performance ([5], [14]).

### 5.3 Linear Matrix Inequality (LMI)-Based Control:

LMIs are commonly used in robust control design to derive conditions that guarantee system stability and performance under input and output constraints. LMIs offer a systematic approach to ensure that the control system remains robust, even under uncertainty ([13], [11]).

## 6. Optimal Control Methods

Optimal control methods aim to minimize a performance index while respecting the system's constraints. The H2 control approach is widely used to minimize the impact of disturbances. The performance index for H2 control is:

$$J = \int_0^{\infty} (x(t)^T Qx(t) + u(t)^T Ru(t))dt \tag{6}$$

where Q and R are positive semi-definite weighting matrices that represent the relative importance of state and control input ([10], [9]).

## 7. Model Predictive Control (MPC)

MPC is a powerful method that optimizes control inputs over a finite prediction horizon, considering future disturbances and constraints. MPC is particularly effective in handling multi-variable constraints. The formulation for MPC is:

$$\min \sum_{k=0}^{N-1} x(k)^T Qx(k) + u(k)^T Ru(k))dt \tag{7}$$

subject to:

$$u(t) \in U, y(t) \in Y \tag{8}$$

where U and Y represent the input and output constraint sets, respectively. MPC is highly effective in real-time applications but requires significant computational resources for large-scale systems ([1], [7]).

## 8. Comparative Analysis of Control Methods

| Control Method | Advantages | Limitations | Applications |
|---|---|---|---|
| LMI-based H2 control | Efficient, ensures stability | May be infeasible for stringent constraints | Robotic arms, spacecraft, inverted pendulum |
| H2/H∞ mixed control | Balances optimal performance and robustness | Complex, challenging to balance norms | Autonomous vehicles, power systems |
| MPC-based fuzzy control | Handles multi-variable constraints directly | Computationally intensive | High-speed trains, automotive systems, wind turbines |

## 9. Conclusions and Future Directions

This paper reviewed key control techniques for T-S fuzzy systems with input and output constraints, focusing on robust, optimal, and predictive control methods. We discussed MPC, anti-windup compensators, and LMI-based methods for ensuring stability and performance under constraints. While these methods are effective, challenges remain in scaling for large systems and high-dimensional applications. Future research should focus on data-driven approaches and the development of adaptive control techniques for handling multiple, concurrent constraints in high-uncertainty environments.

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