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FREQUENCY CONJUGATE ELEMENTS IN MGROUP

Moloy Dutta

Research Scholar, Department of Mathematics, T.M. Bhagalpur University, Bhagalpur

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ABSTRACT

In this paper, I have tried to define Frequency conjugate elements in the context of Multigroup or *M*group. Also, I have provided introduction, historical background, required definitions and results related to this topic that need to be highlighted. These definitions and results will encourage new researchers in the future.

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Introduction:

Set theory was expanded by German mathematician G. Cantor in 19^{th} century. In this type of set which is called an ordinary set, an element occurs only once. Currently another type of set-concept gaining popularity that is called Multiset or *M*set in short.Basic difference between that twoconcepts is,ordinary set does not allow repetition while multiset allow repetition. Also, in ordinary set theory there exist two connections between two elements, either equal or different, while in *M*set theory number of possible connections between two elements is three, maybe they are different, maybe they are same but separate, or maybe they are identically coincided. For instance, in single molecule of nitric acid contains one hydrogen, and one nitrogen and three oxygen atoms. We may also face repetition in the case of factorization a number, storing duplicates elements in sorted manner in C++, digitizing DNA sequence etc. These examples shows that how important this topic is to us and the concept of repetition is more important when objects are present physically.

Literature Review:

After D.E. Knuth and N.G. deBrujin introduced the term 'multiset' to the world of mathematics in 1971, R. R. Yager and J.L. Peterson's contributions to this field were significant. Wayne D Blizard studied several axioms on this topic in 1989. The idea of multiset space was given by S. P. Jena et al. in 2001. D. Sing and J. N. Sing contributed some proofs of combinational properties of Multiset in the year 2003. K.P. Girish & S.J. John brought multisets to the world of mathematics in a completely new way and they also showed us how to define relations, functions in the context of multiset theory.

Many mathematicians have tried to study the properties of algebraic structures on multisets. As a result, the concept of Multiset group has been expanded to us. The name of the mathematicians who are notable in this field are A.M. Ibrahim & P.A. Ejegwa, S.K. Najmul *et al.*, Y. Tella & S. Daniel, P.A. Ejegwa.Theycompleted researches about multiset group and explained it easily to us and also studied different types of results and properties.

S. Debnath & A. Debnath illustratedanddefined*M*rings and worked on some of their important theorems.Suma P and Sunil Jacob John also given a lot of contribution about*M*set relations, *M*set ring, *M*set ideal and also created many results in these concepts.

Prelims:

Definition 1.1 [19]: A multiset or an *M*setY is a collection of some or all elements taken from a crisp set T with their frequency, that is elements with their number of repetitions. This frequency of a member t Є Tis denoted by $Ƒ_Y(t)$.We use $[T]^p$ as *Mset*space, which is the collection of all such *M*sets whose elements are taken from T with maximum number of frequency p.

Example 1.1: Suppose T= {a, b, c} andY= {‹2,a›, ‹4,c›}.

here 'a' appeared 2 times, 'c' appeared 4 times, 'b' appeared 0 times.

So, $Ƒ_Y(a) = 2$, $Ƒ_Y(c) = 4$, $Ƒ_Y(b) = 0$, $Ƒ_Y(t)$ never be negative numbers.

Definition 1.2 [19]: SupposeY, Z Є $[T]^p$ thenYis called a sub *M*set of Z, denoted by Y⊆Z if,$Ƒ_Z(t) \geq Ƒ_Y(t)$ ∀tЄT.

If there exist at least one t Є T with above condition such that,$Ƒ_Y(t)<Ƒ_Z(t)$, then we say that Y is a proper sub*M*set of Z which is denoted by Y⊂Z.

Definition 1.3 [19]: Suppose Y, Z Є $[T]^p$, then YUZand YՈZ are defined as,$Ƒ_{YUZ}(t) =MAX\{ Ƒ_Y(t), Ƒ_Z(t)\}$ and $Ƒ_{Y\cap Z}(t) =MIN\{ Ƒ_Y(t), Ƒ_Z(t)\}$, for all t Є T.

Definition 1.4 [20]: Suppose Yis an *M*set taken from a group (T,⁎)and satisfies the following two conditions:

a.$Ƒ_Y(t⁎r) \geq MIN \{Ƒ_Y(t), Ƒ_Y(r)\}$; for all t,r Є T.

b.$Ƒ_Y(t^{-1}) = Ƒ_Y(t)$ for anyt ЄT.

Then we say that Y is an *M*group taken from (T,⁎).

We will use *MS*[T] and *MG*[T] to representcollection of sets of all *M*sets and*M*groups taken from a group (T,⁎) respectively.

Example 1.2: Let T={1,ɯ, $ɯ^2$}and Y= {<5, 1>,<4, $ɯ^2$>,<4, ɯ>}, where ɯ is an imaginary cube root of unity, T will forma group with respect to the usual multiplication. Also, we can verify that Y will form an *M*group by following table.

.	1 (5)	ɯ (4)	$ɯ^2$ (4)
1 (5)	1 (5)	ɯ (4)	$ɯ^2$ (4)
ɯ (4)	ɯ (4)	$ɯ^2$ (4)	1 (5)
$ɯ^2$ (4)	$ɯ^2$ (4)	1 (5)	ɯ (4)

Theorem 1.1 [22]: Suppose Y Є*MG*[T], where (T,∗) is a group with identity element e then,(a) $Ƒ_Y(e) \geq Ƒ_Y(t)$ (b) $Ƒ_Y(t^p) \geq Ƒ_Y(t)$ for anyt Є T

Proof:(a) $Ƒ_Y(e) = Ƒ_Y(t*t^{-1}) \geq MIN\{Ƒ_Y(t), Ƒ_Y(t^{-1})\} = MIN\{Ƒ_Y(t), Ƒ_Y(t)\}$

$= Ƒ_Y(t)$

(b) we can use mathematical induction to prove this.

Definition 1.5 [22]: Let YЄ*MS*[T]where (T,∗) is a group then,

Y^{-1} Є *MS*[T] and$Ƒ_Y{}^{-1}(t) = Ƒ_Y(t^{-1})$.

Definition 1.6 [22]: SupposeYЄ*MG*[T] where (T,∗) is a group. Then Z Є*MS*[T] issaid to be a sub *M*group of Y, if Z⊆YandZ Є *MG*[T].

Theorem 1.2 [22]: Y Є *MG*[T] ⟺$Ƒ_Y(t*r^{-1}) \geq MIN\{Ƒ_Y(t), Ƒ_Y(r)\}$,

∀t, r Є T.where (T,∗) is a group.

Proof: Suppose, Y Є *MG*[T]

Then,$Ƒ_Y(t * r^{-1}) \geq MIN\{Ƒ_Y(t), Ƒ_Y(r^{-1})\} = MIN\{Ƒ_Y(t), Ƒ_Y(r)\}$ ∀ t, r Є T.

For converse case,

$Ƒ_Y(t*r^{-1}) \geq MIN\{Ƒ_Y(t), Ƒ_Y(r)\}$ holds ∀ t, r Є T.

Now, $Ƒ_Y(e) = Ƒ_Y(t*t^{-1}) \geq MIN\{Ƒ_Y(t), Ƒ_Y(t)\} = Ƒ_Y(t)$ for all t ЄT

Therefore, $Ƒ_Y(e) \geq Ƒ_Y(t)$ for all t ЄT

Now$Ƒ_Y(t) = Ƒ_Y(e*t) \geq MIN\{Ƒ_Y(e), Ƒ_Y(t^{-1})\}$

$= Ƒ_Y(t^{-1}) = Ƒ_Y(e*t^{-1}) \geq MIN\{Ƒ_Y(e), Ƒ_Y(t)\} = Ƒ_Y(t)$ ∀ t Є T.

So, $Ƒ_Y(t) = Ƒ_Y(t^{-1})$.

$Ƒ_Y(t*r) = Ƒ_Y(t*(r^{-1})^{-1}) \geq MIN\{Ƒ_Y(t), Ƒ_Y(r^{-1})\} = MIN\{Ƒ_Y(t), Ƒ_Y(r)\}$ ∀ t, r Є T.

Hence Y is an *M*group.

Definition 1.7 [20]:Y∈*MG*[T] where (T,∗) is a group, is called an abelian *M*groupover T if for any t,r ∈ T, $Ƒ_Y(t*r) = Ƒ_Y(r*t)$.

AMG[T] is the Set of all possible abelian *M*groups over T.

Definition 1.8[20]:Y∈*MG*[T] where (T,∗) is a group, is called a normal *M*groupif$Ƒ_Y(t*r*t^{-1}) \geq Ƒ_Y(r)$ for anyt, r ∈ T.

Definition 1.9[54]: Y∈*MG*[T] where (T,∗) is a group then Madhya of Y is denoted by $\mathcal{M}$(Y) is an *M*setwhich is defined as,

$Ƒ_{\mathcal{M}(Y)}(t)=0$, where $Ƒ_Y(t*r) \neq Ƒ_Y(r*t)$ for some r ∈ T.

$Ƒ_{\mathcal{M}(Y)}(t)= Ƒ_Y(t)$, where $Ƒ_Y(t*r) = Ƒ_Y(r*t)$ ∀r ∈ T.

Note -It is very easy to show that if Y∈*MG*[T]then,□(Y)⊆Y.

Theorem 1.3[54]:Let Y∈*MG*[T] where (T,∗) is a group then,

e $\in^r$□(Y)where r is the positive frequency.

Proof: It is very easy to prove.

Theorem 1.4[54]:Let Y∈*MG*[T] where (T,∗) is a group then,

If t ∈ □(Y)with frequency greater than zero

⇒t^{-1} ∈ □(Y) with frequency greater than zero.

Proof: As t ∈ □(Y), $Ƒ_Y(t*r^{-1}) = Ƒ_Y(r^{-1}*t)$ for any r^{-1} of Y

⇒ $Ƒ_Y((t*r^{-1})^{-1}) = Ƒ_Y((r^{-1}*t)^{-1}) \Rightarrow Ƒ_Y(r*t^{-1}) = Ƒ_Y(t^{-1}*r)$

So,t^{-1}∈ □(Y) and $Ƒ_{\mathcal{M}(Y)}(t^{-1})=Ƒ_Y(t^{-1})= Ƒ_Y(t)=Ƒ_{\mathcal{M}(Y)}(t)> 0$ [Hence proved]

Theorem 1.5[54]: Let Y∈*MG*[T] where (T,∗) is a group and if t,r ∈ □(Y)with positive frequency then, t∗r ∈ □(Y)with positive frequency.

Proof: We can prove it very easily.

Theorem 1.6[54]:Let Y∈*MG*[T] where (T,∗) is a group then

□(Y) is a sub *M*group of Y.

Proof: □(Y)is a sub *M*setof Y.So, wewill only cheque, it will satisfy necessary & sufficient condition of *M*group or not.

Suppose, Ƒ$_{\mathcal{M}(Y)}$(t) = 0 orƑ$_{\mathcal{M}(Y)}$(r) = 0 for some t, r Є T

Then clearly, Ƒ$_{\mathcal{M}(Y)}$($t*r^{-1}$)≥*MIN*{Ƒ$_{\mathcal{M}(Y)}$(t), Ƒ$_{\mathcal{M}(Y)}$(r)}.

Now if, Ƒ$_{\mathcal{M}(Y)}$(t) ≠ 0 &Ƒ$_{\mathcal{M}(Y)}$(r)≠ 0 for some t, r Є T

Ƒ$_{\mathcal{M}(Y)}$(t) = Ƒ$_Y$(t) > 0 &Ƒ$_{\mathcal{M}(Y)}$(r)= Ƒ$_Y$(r) >0

⇒Ƒ$_{\mathcal{M}(Y)}$(t^{-1})> 0 &Ƒ$_{\mathcal{M}(Y)}$(r^{-1})>0

⇒Ƒ$_{\mathcal{M}(Y)}$($t*r^{-1}$) >0. [By previous theorem]

Ƒ$_{\mathcal{M}(Y)}$($t*r^{-1}$)≥*MIN*{Ƒ$_Y$(t), Ƒ$_Y$(r)}

= *MIN* {Ƒ$_{\mathcal{M}(Y)}$(t),Ƒ$_{\mathcal{M}(Y)}$(r)}

Hence Ƒ$_{\mathcal{M}(Y)}$($t*r^{-1}$) ≥*MIN*{Ƒ$_{\mathcal{M}(Y)}$(t), Ƒ$_{\mathcal{M}(Y)}$(r)} for all t, rЄ T.

Therefore, □(Y) is a sub *M*group of Y.

Definition 1.9.1[54]:Let YЄ*MG*[T] where (T,∗) is a group then Madhya□$_□$(□)of an element t Є T in Y is an *M*setdefined as,

***F*$_{□_□(□)}$(r)=0, where r Є T and Ƒ$_Y$(t∗r) ≠ Ƒ$_Y$(r∗t).**

***F*$_{□_□(□)}$(r)= Ƒ$_Y$(r), where r Є T and Ƒ$_Y$(t∗r) = Ƒ$_Y$(r∗t).**

Theorem 1.7 [54]:Let YЄ*MG*[T] where (T,∗) is a group then ealways belongs to □(Y) with positive frequency.

Proof: It is very easy to prove.

Maindefinitionswithproperties:

Definition2.1:Frequency conjugate element: Let YЄ*MG*[T] where (T,∗) is a group,then an element r□T is said to be Ƒ conjugateor frequency conjugate tot□T, if there exist an element g in T such that, Ƒ$_Y$(r) = Ƒ$_Y$($g*t*g^{-1}$).

Definition 2.2: Let YЄ*MG*[T] where (T,∗) is a group then for t □T,Cl$_Y$(t)is an *M*set such that,*F*$_{Cl_Y(t)}$(r)= Ƒ$_Y$(r) if r is conjugate to t otherwise *F*$_{Cl_Y(t)}$(r)= 0.

Theorem 2.1:Let YЄ*MG*[T] where (T,∗) is a group.If t is Ƒ conjugate to r then t^{-1} is also Ƒ conjugate to r^{-1}.

Proof: Since t is Ƒ conjugate to r,

Therefore, Ƒ$_Y$(t) = Ƒ$_Y$($g*r*g^{-1}$) for some g in T.

So,Ƒ$_Y$(t^{-1}) = Ƒ$_Y$($(g*r*g^{-1})^{-1}$)⇒Ƒ$_Y$(t^{-1}) = Ƒ$_Y$($(g^{-1})^{-1}*r^{-1}*g^{-1}$)

⇒Ƒ$_Y$(t^{-1}) = Ƒ$_Y$($g*r^{-1}*g^{-1}$).

Hence, t^{-1} is Ƒ conjugate to r^{-1}.

Theorem2.2: Let Y€*MG*[T] where (T,∗) is a group.

If t is Ƒconjugate to r then t^{-1} is also Ƒconjugate to r.

Proof: Since t is Ƒ conjugate to r, Ƒ$_Y$(t) = Ƒ$_Y$($g*r*g^{-1}$) for some g in T.

⇒Ƒ$_Y$(t^{-1}) = Ƒ$_Y$($g*r*g^{-1}$).Hence, t^{-1} is Ƒ conjugate to r.

Theorem 2.3: Let(T,∗) is a group, Y€*MG*[T], and Ƒ$_Y$(u) is positive for an element u € T,then u € □□$_□$(□)with positive frequency.

Proof: Clearly element u is Ƒ conjugate to u here because,

Ƒ$_Y$(u) = Ƒ$_Y$($e*u*e^{-1}$), where e is the identity element of T.

So, $F_{Cl_Y(u)}$(u)= Ƒ$_Y$(u), which is positive.

Theorem 2.4: Let Y€*MG*[T] where (T,∗) is a group then □□$_□$(□)is a Sub *M*set of Y.

Proof: If t **€** □□$_□$(□)with positive frequency.Then, $F_{Cl_Y(u)}$(t)= Ƒ$_Y$(t).

Now if t **€** □□$_□$(□) with zero frequency then two cases arise,

Case-1: If t is not F conjugate to**u** but Ƒ$_Y$(t) positive.

Then obviously, $F_{Cl_Y(u)}$(t) ≤Ƒ$_Y$(t).

Case-2: If t is not F conjugate to**u** and Ƒ$_Y$(t) = 0.

Then, $F_{Cl_Y(u)}$(t) =Ƒ$_Y$(t).

Therefore, for all t **€ T,** $F_{Cl_Y(u)}$(t) ≤Ƒ$_Y$(t). [Hence proved.]

Theorem 2.5: Let Y€*MG*[T] where (T,∗) is a group. If t, rare two elements of □(Y) with positive and same frequency then t is Ƒ conjugate to r.

Proof: Here, Ƒ$_Y$(r) = Ƒ$_Y$(t)⇒Ƒ$_Y$($r*t^{-1}*$**t**) = Ƒ$_Y$(t)⇒Ƒ$_Y$($(r*t^{-1})*$**t**) = Ƒ$_Y$(t).

$\Rightarrow Ƒ_Y(t*(r*t^{-1})) = Ƒ_Y(t)$ [since t is the element of □**(Y)]**

$\Rightarrow Ƒ_Y(t*r*t^{-1}) = Ƒ_Y(t)$. [Hence proved]

Future possible works: Considering the importance of this topic we can study and create some other results on frequency conjugate elements in future. We can provesome results in future like, if t and s are Ƒ conjugate to r then t∗s is also Ƒ conjugate to r or not, Frequency of t∗s in $Cl_Y(u)$,$Cl_Y(u)$ forms an *M*group or not,$Cl_Y(u)$ forms a normal sub *M*group of Y or not etc.

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