



A Brief Discussion on Flow of a Viscous Lubricant in a Slider Bearing

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ABSTRACT

In this paper we study about flow of a viscous lubricant in a slider bearing. In fluid dynamics, there has been a growing interest in the use of this method to the problem of fluid flow. Investigators like Martius Tong and Fung Norrie D.H. and varies G. De, Doctors, Atkinson and Gelder have contributed a lot in the area.



1. Introduction:

Viscosity is the resistance of a fluid to flow when a fluid is flowing, the molecules comprising it experience friction due to the molecular interaction among them. It means that different speeds when the same force is applied. Another way to look at it is that the force required to induce movement of the fluid will be large for more viscous fluids.

In other words, the shear stress developed in the fluids is a function of viscosity. The fluids in which the shear stress is proportional to the shear rate, are called Newtonian fluids and the constant of proportionality is called viscosity.

2. Flow of a Viscous Lubricant in a Slider Bearing

The slider bearing consists of a short sliding pad moving at a velocity $u = v_0$ relative to a stationary pad inclined at a small angle with respect to the stationary pad, and that small gap between the two pads is filled with a lubricant. Since the ends of the bearing are generally open, the pressure there is atmospheric, P_0 . If the upper pad is parallel to the base plate, the pressure everywhere in the gap must be atmospheric (because dp/dx is a constant for flow between parallel plates) and the bearing cannot support any transverse load. If the upper pad is inclined to the base pad, a pressure distribution in general, a function of x and y is set up in the gap for large values of v_0 , the pressure generated can be of sufficient magnitude to support heavy loads normal to the base pad.[1]

When the width of the gap and the angle of inclination are small, we may assume that $y = v_0$ and the pressure is not a function of y . Assuming a two-dimensional state of flow and a small angle of inclinations, and neglecting the normal stress gradient (in comparison with the shear stress gradient), the equation governing the flow of the lubricant between the pads can be reduced to



$$\mu \frac{\partial^2 v_x}{\partial y^2} = \frac{dp}{dx}, 0 < x < L \quad \dots \dots \dots (3.1)$$

Where the pressure gradient is given by

$$\frac{dp}{dx} = \frac{\sigma \mu v_0}{h^2} \left(1 - \frac{H}{h} \right), H = \frac{2h_1 h_2}{h_1 + h_2} \quad \dots \dots \dots (3.2)$$

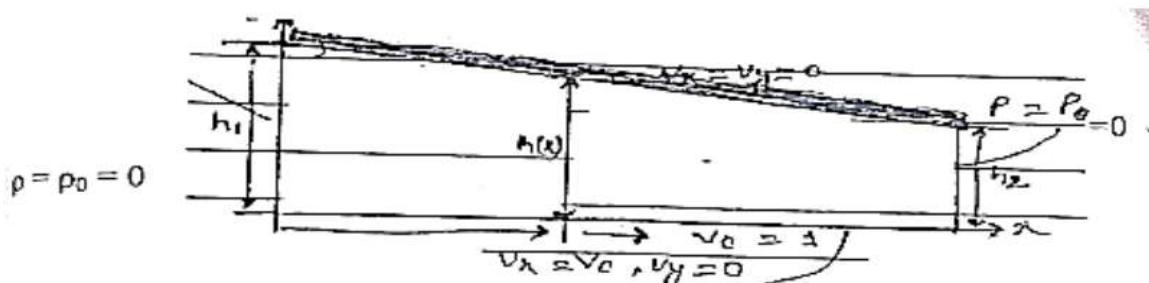


Figure (a)

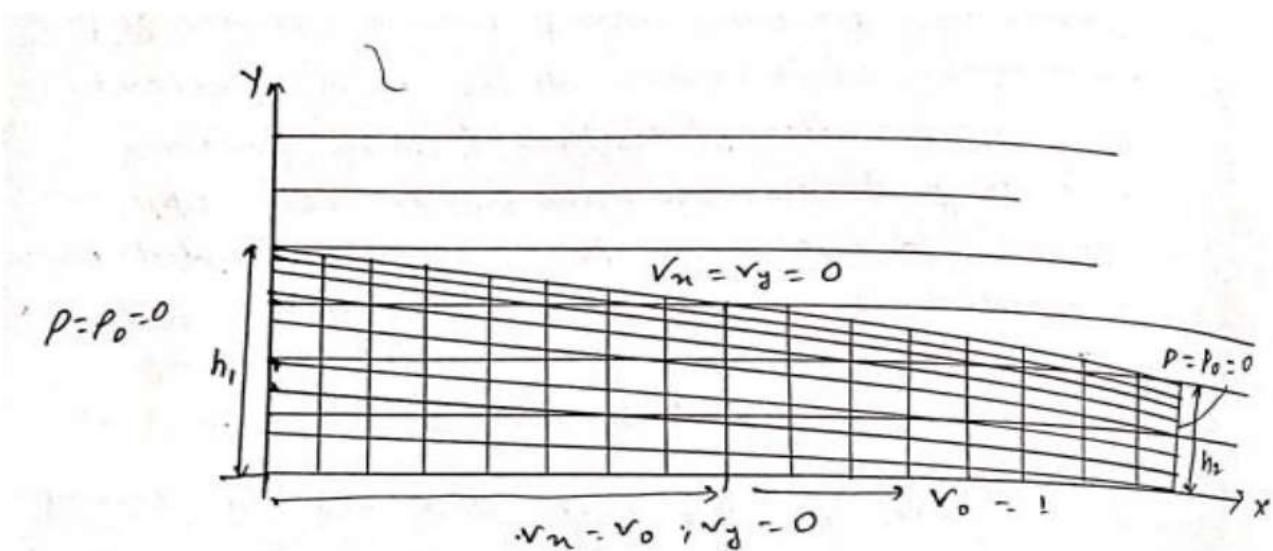


Fig. (b)

Figure schematic and the finite element mesh for slider bearing.



The solution of equation (3.1) and (3.2) subject the boundary conditions.

$$u_x(0,0) = V_0, u_x(x, h) = 0 \quad \dots \dots \dots (3.3)$$

$$\text{is } v_x(x, y) = \left(v_0 - \frac{h^2 dp}{2\mu dxh} \right) \left(1 - \frac{y}{h} \right) \quad \dots \dots \dots (3.4)$$

$$p(x) = \frac{\sigma \mu v_0 L (h_1 - h)(h - h_2)}{h^2 (h_1^2 - h_2^2)} \quad \dots \dots \dots (3.5)$$

$$\sigma_{xy}(x, y) = \mu \frac{\partial v_x}{\partial y} = \frac{dp}{dx} \left(y - \frac{b}{2} \right) - \mu \frac{v_0}{h} \quad \dots \dots \dots (3.6)$$

$$\text{where } h(x) = h_1 + \frac{h_2 - h_1}{L} x \quad \dots \dots \dots (3.7)$$

In the finite element analysis we do not make any assumption concerning v_y and the pressure gradient, and solve the stokes equation with the following choice of parameters $h_1 = 2h_2 = 8 \times 10^{-4} \text{ ft}$, $L = 0.36 \text{ ft}$, $\mu = 8 \times 10^{-4} \text{ lb s / in}^2$, $v_0 = 30 \text{ ft} \quad \dots \dots \dots (3.8)$

We use a mesh (Mesh 1) of 18×8 linear quadrilateral elements to analyse the problem. The mesh and boundary conditions are shown in fig (b). The penalty parameter is chosen to be $\gamma = \mu \times 10^8$. Table A contains a comparison of the finite element solutions and analytical solutions for the velocity pressure and shear stress fig (c) contains plots of the horizontal velocity v_x at $x = 0, 0.18$ and 0.36 ft fig (d) contains plots of pressure and shear stress as a function of x at $y = 0$. The finite element solutions for the pressure and shear stress were computed at the centre of the first row of elements along the moving block. The results are good agreement with the analytical solution (3.1) and (3.3) validating the assumptions made in the development of the analytical solution.

Comparison of finite element solution velocities with the analytical solutions for viscous fluid in a slider bearing.

Table A

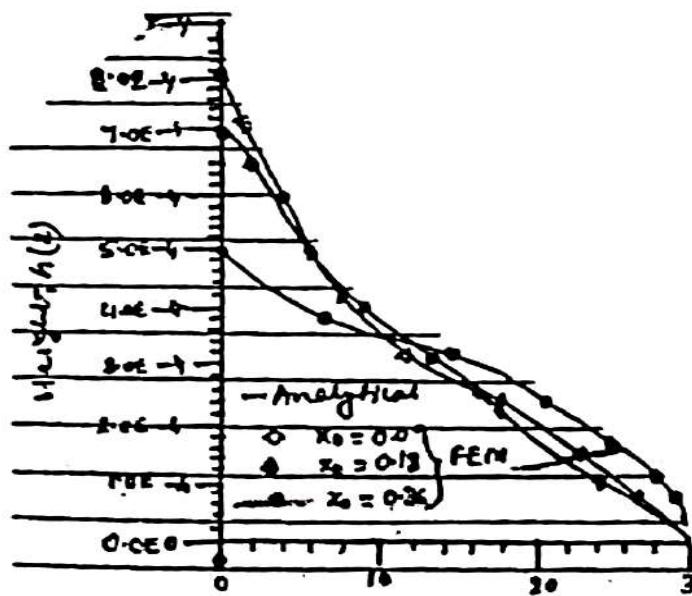
\bar{y}	$v_x(0, y)$			$U_x(0, 1B, y)$			$v_x(0.36, y)$	
	FEM	Analy.	\bar{y}	FEM	Analy.	\bar{y}	FEM	Analy.
0.0	30.000	30.000	0.00	30.000	30.000	0.00	30.000	30.000
1.0	22.923	22.969	0.75	25.139	25.156	0.50	29.564	29.531
2.0	16.799	16.875	1.50	20.596	20.625	1.00	28.182	28.125
3.0	11.626	11.719	2.25	16.372	16.406	1.50	25.853	25.781
4.0	7.403	7.500	3.00	12.465	12.500	2.00	22.577	22.500



5.0	4.130	4.219	3.75	8.814	8.906	2.50	18.354	18.281
6.0	1.805	1.875	4.50	5.600	5.625	3.00	13.184	13.125
7.0	0.429	0.469	5.25	2.642	2.656	3.50	7.066	7.031
8.0	0.000	0.000	6.00	0.000	0.00	4.00	0.000	0.000

x	Analytical solution		FEM Solution			
	$\bar{p}(x,0)$	$-\sigma_{xy}(x,0)$	\bar{x}	\bar{y}	\bar{p}	$-\sigma_{xy}$
0.01	7.50	59.99	0.1125	0.4922	8.46	56.61
0.03	22.46	59.89	0.3375	0.4766	25.46	56.60
0.05	37.29	59.67	0.5625	0.4609	42.31	56.47
0.07	51.89	59.30	0.7875	0.4453	58.76	56.17
0.09	66.12	58.77	1.0125	0.4297	74.69	55.69
0.27	129.60	38.46	2.5875	0.3203	134.40	41.77
0.29	118.57	32.71	2.8125	0.3047	125.60	36.93
0.31	99.58	25.70	3.0375	0.2891	107.60	30.76
0.33	70.30	17.04	3.2625	0.2734	77.39	22.89
0.35	27.61	6.31	3.4875	0.2878	30.80	12.82

$$\bar{x} = 10x, \bar{y} = y \times 10^4, \bar{p} = px \times 10^{-2}$$



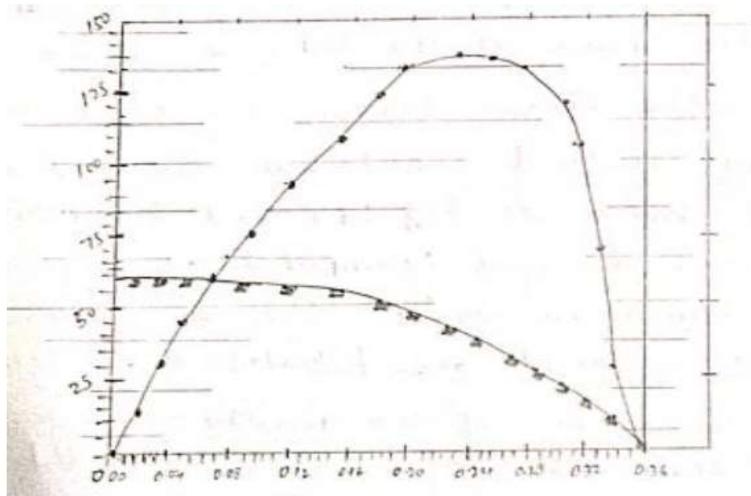
Velocity $v_x(x_0, y)$ **Figure (c)** velocity distribution for the slider bearing problem

Figure (d) pressure and shear stress distribution for the slider bearing problem

Lid Driven Cavity Flow

Consider the Laminar flow of a viscous, incompressible fluid in a square cavity bounded by three motionless walls and a lid moving at a constant velocity in its own plane (see figure (e)) singularities exist at each corner where the moving lid meets a fixed wall. This example is one that has been extensively studied by analytical, numerical and experimental methods, and it is often used as a benchmark problem to test a new numerical method or formulation.

Assuming a unit square and a unit velocity of the top wall, we can disc retire the flow region using a uniform 8×8 mesh of linear elements or 4×4 of nine-node quadratic elements. At the singular points, namely at the top corner of the lid, we assume that $v_x(x, 1) = v_0 = 1.0$. The linear solution for the horizontal velocity along the vertical centreline obtained with the two meshes shown is figure (f) and the variation of pressure along the top wall (computed at the reduced Gauss points) is shown in figure(g). The numerical values of the velocity field are tabulated in table B. It is clear that the value of the penalty parameter between $\gamma = 10^2$ and 10^8 has a small effect on the accuracy of the solution figure(h) contains plots of the centre velocity $u_x(0.5, y)$ as a function of y for 8×8 and 16×20 meshes of bilinear elements.



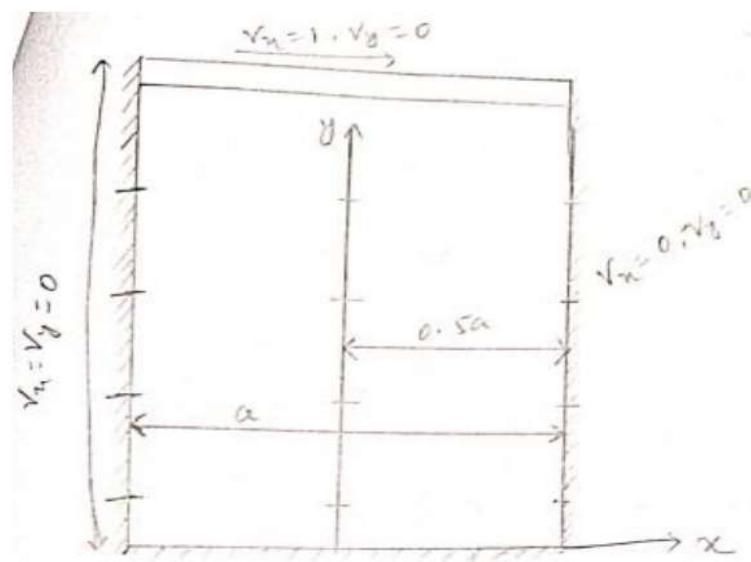


Figure (e): Boundary condition for Lid-driven cavity problem

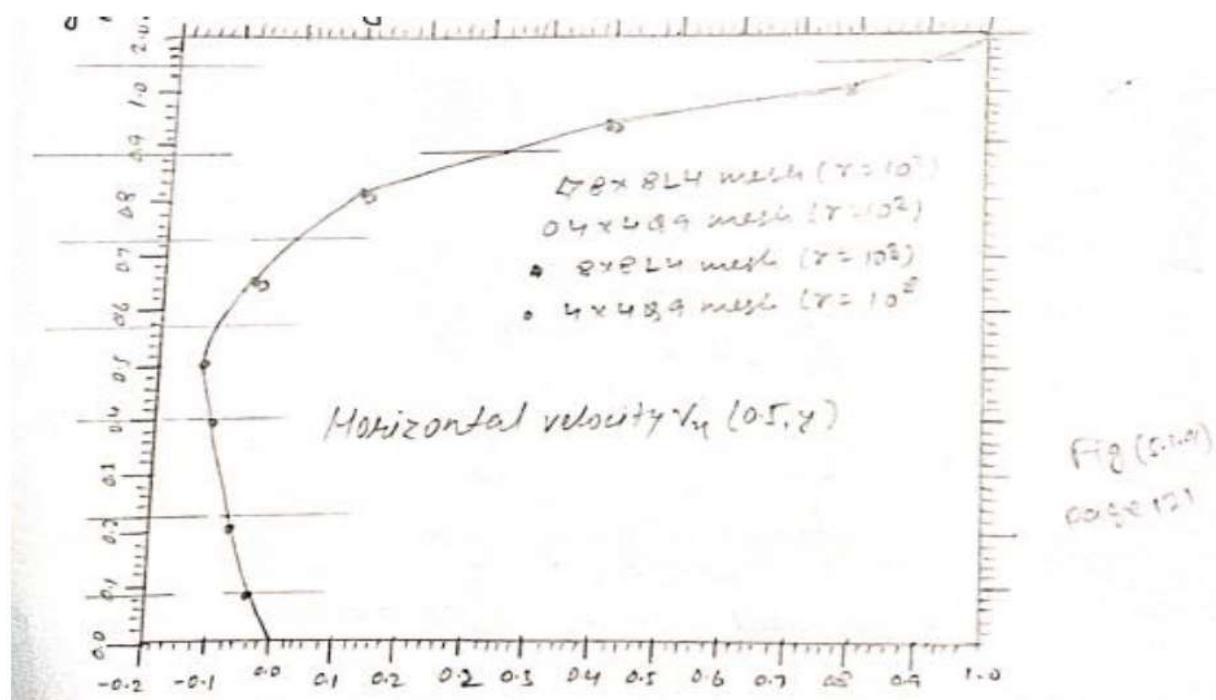
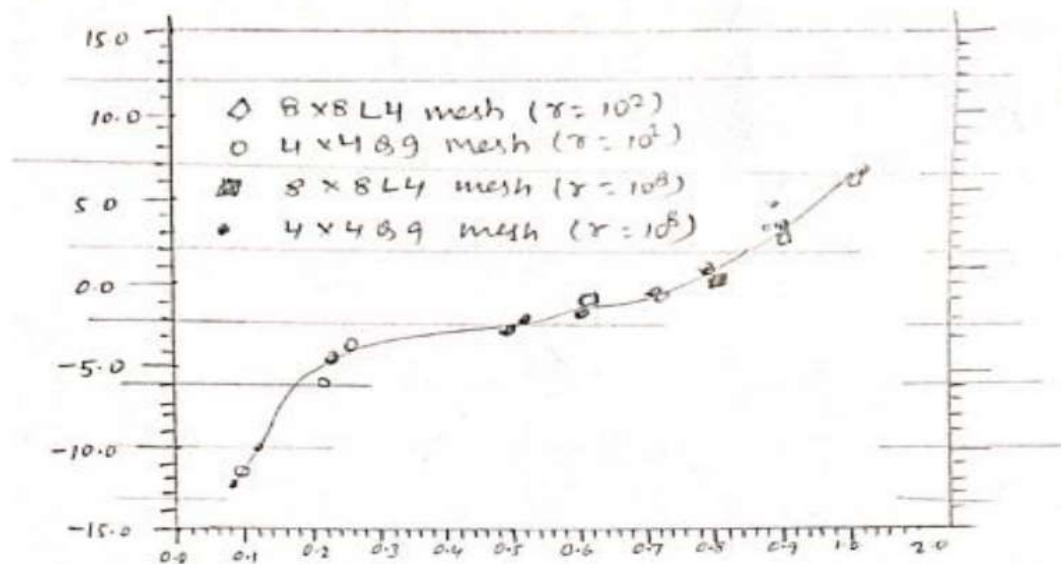
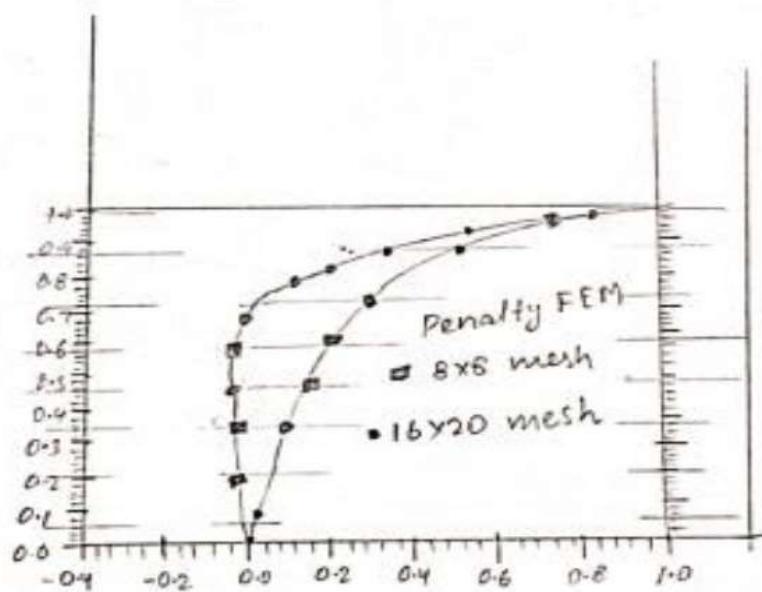


Figure (f): Plots of horizontal velocity $v_x(0.5, y)$ along the vertical centreline of the cavity.

Figure (g): plots of pressure $P(x, y)$ along the top wall of the cavityHorizontal velocity, $v_x(0.5y)$ Figure (h): velocity $v_x(0.5, y)$ versus y for 8×8 and 16×20 meshes of bilinear elements.

**Table – B**

Velocity $v_x(0.5, y)$ obtained with various values of the penalty parameter γ .

y	Mesh, 8×8 L4		Mesh, 4×4 Q9	
	$\gamma = 10^2$	$\gamma = 10^8$	$\gamma = 10^2$	$\gamma = 10^8$
0.125	-0.0557	-0.0579	-0.0589	-0.0615
0.250	-0.0938	-0.0988	-0.0984	-0.1039
0.375	-0.1250	-0.1317	-0.1320	-0.1394
0.500	-0.1354	-0.1471	-0.1442	-0.1563
0.625	-0.0818	-0.0950	-0.0983	0.1118
0.750	0.0958	0.0805	0.0641	0.0481
0.875	0.4601	0.4501	0.4295	0.4186

Reference :

1. B. P. Leonard Survey of finite differences Of opinion on numerical modeling of the incompressible defective convection equation. In T.J.R. Hughes, editor, Finite element Method for convection Dominated Flows ASME, AMD 1979.
2. Ph.D. Thesis: Application of F.E. Theory of viscous fluid flow, M.U. 1990
Doctors, L.J.: An application of finite element technique to boundary value problems of potential flow Int. Jour.: Num. Meth. Engg. 2, 243-52 (1970)



3. Curle. N and Davis. H.J: "Modern fluid Dynamics vol. I.D. Van and Nostrand comp. Ltd. 1968. Aris. R.: Vector tensors and the Basic equations of fluid mechanics, Prentice Hall. Englewood, New Jersey, 1962
4. Hamming, R.W.: Numerical methods for scientists and Engineering McGraw Hill New York (1962)