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Intuitionistic L-Fuzzy Sets of Third Type: Extending Fuzzy Logic with Refined Uncertainty Measures

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ABSTRACT

In recent years, the field of fuzzy sets has expanded significantly, with various extensions being proposed to handle different types of uncertainty. This paper introduces a novel extension of the classical fuzzy sets, referred to as Intuitionistic L-Fuzzy Sets of Third Type (ILFSTT)**.** We explore the definition and basic properties of ILFSTT, including operations such as union, intersection, complement, and subset relationships. This new type of fuzzy set provides a more refined model of uncertainty compared to traditional fuzzy sets and intuitionistic fuzzy sets. Furthermore, we establish some key results related to the algebraic structure of ILFSTTs, which can have potential applications in areas requiring advanced fuzzy logic models.

1. Introduction

Intuitionistic fuzzy sets, introduced by Krassimir Atanassov in 1983, are an extension of fuzzy sets that handle not only membership but also non-membership of elements in a set. These sets also introduce a concept of uncertainty, which is the difference between 1 and the sum of membership and non-membership. Since their introduction, intuitive fuzzy sets and their various extensions, such as Intuitionistic L-Fuzzy Sets and Intuitionistic L-Fuzzy Sets of Second Type (ILFSST), have found applications in areas such as decision-making, control systems, and artificial intelligence.

In this paper, we introduce the Intuitionistic L-Fuzzy Sets of Third Type (ILFSTT), which further extends the theory of intuitionistic fuzzy sets. We define the basic properties of ILFSTT and explore the fundamental operations associated with them. This new class of fuzzy sets provides an additional layer of complexity that can be utilized in applications that require a more nuanced approach to uncertainty.

2. Materials and Methods

2.1. Basic Definitions

- **Definition 2.1 (Intuitionistic Fuzzy Set)**:

  Let X be a non-empty set. An **Intuitionistic Fuzzy Set (IFS)** A in X is defined as an object of the form

$$\mathbf{A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}}$$

  Where, $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership and non-membership of x in A, respectively, and $\mu_A(x) + \nu_A(x) \leq 1$ for all in $x \in X$. The uncertainty of an element x is defined as $1 - \mu_A(x) - \nu_A(x)$ [1], [2].

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- **Definition 2.2 (Intuitionistic L-Fuzzy Set)**:

  An Intuitionistic L-Fuzzy Set (ILFS) A in a universal set E is defined as an object of the form

$$\mathbf{A = \{\langle x,\mu_A(x),\nu_A(x)\rangle | x\in E\}}$$

  Where, $\mu_A(x)$ and $\nu_A(x)$ denote the degree of membership and non-membership of x in A, and L represents an unary involute order-reversing operation [3].

- **Definition 2.3 (Intuitionistic L-Fuzzy Set of Second Type)**: An Intuitionistic L-Fuzzy Set of Second Type (ILFSST) A in a universal set E is defined as

$$\mathbf{A= \{\langle x,\mu_A(x),\nu_A(x)\rangle | x\in E\}}$$

  With a defined unary involute order-reversing operation and additional structural properties [3].

2.2. Intuitionistic L-Fuzzy Set of Third Type (ILFSTT)

- **Definition 3.1 (Intuitionistic L-Fuzzy Set of Third Type)**: An Intuitionistic L-Fuzzy Set of Third Type (ILFSTT) A in a universal setE is defined as

$$\mathbf{A=\{\langle x,\mu A(x),\nu A(x)\rangle | x\in E\}}$$

  Where, $\mu_A(x)$ and $\nu_A(x)$ are the membership and non-membership functions, and L is an involute complement operation. The uncertainty is defined as $1-\mu_A(x)-\nu_A(x)$, providing a deeper layer of uncertainty compared to previous types [3].

3. Results and Discussions

3.1. Properties of ILFSTT

The key operations for Intuitionistic L-Fuzzy Sets of Third Type include the union, intersection, complement, and subset relations.

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- **Example 3.1**:
  Let X={1,2,3,4} and define the set A={⟨1,0.8,0.2⟩,⟨2,0.6,0.4⟩,⟨3,0.9,0.1⟩,⟨4,0.7,0.3⟩}. The membership and non-membership values of the elements are as defined above.
- **Example 3.2**:

  Let X={a,b,c,d} and let A and B be two ILFSTTs. The following relationships hold:

  - (a) $A \not\subset B$ since $\mu_A(x) \neq \mu_B(x)$ for $x = c$ and $\nu_A(x) \neq \nu_B(x)$ for $x = c, d$.
  - (b) $B \not\subset A$ since $\mu_B(x) \neq \mu_A(x)$ for $x = a, b, d$ and $\nu_B(x) \neq \nu_A(x)$ for $x = a, b$.
  - (c) $A \neq B$ because $\mu_A(x) \neq \mu_B(x)$ and $\nu_A(x) \neq \nu_B(x)$ for certain elements.

3.2. Algebraic Operations

Several algebraic operations such as **union**, **intersection**, and **complement** are explored and defined in terms of their set-theoretic properties. The operations follow specific rules and can be defined symbolically as:

- (a) $A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle\}$
- (b) $A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle\}$

4. Conclusions

In this paper, we introduced the **Intuitionistic L-Fuzzy Sets of Third Type (ILFSTT)** and studied several fundamental operations on them. These operations, including union, intersection, subset relations, and complement, provide a comprehensive algebraic structure for these sets. The introduction of this third type of fuzzy set allows for more complex representations of uncertainty, which could have significant applications in various fields that require advanced fuzzy logic models. Future work will include exploring the deeper applications of ILFSTTs and their potential in solving real-world problems.

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