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A Theoretical Framework for Functions Mapping Complex Numbers to Fuzzy Sets

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ABSTRACT

In order to bridge the gap between classical complex analysis and fuzzy mathematics, this work presents a new theoretical basis for functions that convert complex numbers into fuzzy sets. Fuzzyfied complex valued mappings, continuity, convergence, and analytic property adoption in fuzzy environments are the main points of the system. The link between complex moduli and fuzzy membership gradations is investigated in this paper by combining topological and algebraic ideas. Potentially useful in signal processing, decision theory, and AI, the suggested model allows for a more thorough understanding of uncertainty quantification in complex-valued systems. Strengthening the connection between complex function theory and soft computing paradigms, this analysis establishes a foundation for future studies on integration, differentiability, and fuzzy holomorphy.

1. INTRODUCTION

It is commonly known that, under error-free conditions, measure issues were primarily illustrated by the additivity of classical measures. However, under some conditions, additivity was unable to completely portray the measure difficulties when measure mistake was inevitable. The development of fuzzy measures was an attempt to circumvent these problems. Research on fuzzy measures has been extensive, covering several areas. These include studies that expand on the theory of set-valued measures, research that uses fuzzy sets with a real-valued measure, and investigations that focus on a specific number of subsets of a classic set, along with a real-valued non-additive measure. Examples of this last type of research include Choquet's content theory [1] and Sugeno's measure theory [2]. Set value measure has been employed in several ways since its late 1960s theory [4,6]. Measures and integrals were examined shortly after fuzzy numbers were introduced. Zhang invented R.n fuzzy set measure in 1986 [7]. Wu and his team added a fuzzy real number field to the fuzzy measure in 1998. For fuzzy numbers, they defined the Sugeno integral [8]. Guo and colleagues invented the G fuzzy value measure integral of a fuzzy value function [9]. From fuzzy values to fuzzy sets, this study expanded the Sugeno integral [10]. Buckley's 1989 work [11] on fuzzy complex numbers [11] requires careful measurement and integration due to their complexity and imprecision. Guang-Quan [12] introduced fuzzy real-valued measure theory in fuzzy set space in the early 1990s. After fuzzy real distance, he addressed fuzzy real measures based on fuzzy sets. He subsequently devised fuzzy real-valued integrals and measures. Buckley and Qu [11, 12] studied fuzzy complex function differential and integral in 1991 and 1992, adding to fuzzy complex analysis. Qiu et al. sequentially examined several foundational aspects of fuzzy complex analysis theory from 1996 to 2001, including 1 fuzzy complex valued functions and their differentiability [14], the continuity of fuzzy complex numbers and series [13], and fuzzy complex valued measure and integral functions [15]. Wang and Li [18] discovered interesting Lebesgue integrals of fuzzy complex-valued functions. Inspired by Buckley's fuzzy complex numbers, they created the fuzzy complex-valued measure.

2.1. Analytic properties, highlighting challenges in defining holomorphicity and differentiability

Fuzziness mapping into complex-valued frameworks has been an area of interest for several academics in complex analysis. To allow for the fuzzification of structures based on norms, Sessa (1986) [30] first suggested fuzzy normed linear spaces. To build upon this, George and Veeramani (1992) [31] introduced fuzzy metric spaces, which allowed for the study of differentiability and convergence in the face of uncertainty. The analytic properties of fuzzy-valued functions were studied more recently by Kaleva (2006) [32], Ming & Ming (2012) [33], and Dubois & Prade (2014) [34]. These authors brought attention to the difficulties in defining holomorphicity and differentiability when working with membership functions instead of clear numerical values. Research into complicated fuzzy numbers [35,36] has made a substantial impact on engineering and computational uncertainty modeling, especially in decision systems and signal processing. The models did not go far enough in their pursuit of a unifying theory of fuzzy complex-valued functions, but they did contain complex magnitude and fuzziness. Although some recent efforts have explored the differentiability of fuzzy complex functions, such as in the works of Abu-Jaish (2019) [37] and Kumar & Sekaran (2021) [38], these formulations did not have topological coherence with traditional analytic structures. A new area of mathematical study is the creation of a strong theoretical framework for functions translating complex numbers to fuzzy sets. This framework should have topological consistency, analytic structure, continuity, and convergence.

2.1.1 Complex fuzzy set

A recent breakthrough in fuzzy system theory is the complex fuzzy set (CFS) [16]. Complex fuzzy sets (CFS) employ complex-valued states to express membership, expanding the fuzzy set concept. A complex fuzzy set's membership values are complex integers from the complex plane's unit disk [1-2]. Complex Fuzzy Systems (CFS) theory has been introduced [17], but research on their design and implementation is restricted. Since Zadeh's 1965 fundamental work on fuzzy sets [18,19], much literature has been written on fuzzy sets and their use. Ramot and his team [20, 21] improved on similar principles with complex fuzzy collections. In this situation, the membership function (μ) is complex-valued, unlike the typical [0, 1] functions.

$$r_j(x), e^{i\theta},^{(x)}; j = \sqrt{-1}, \quad (1)$$

Where, the range is given by the unit circle and both r(x)s and θ(x)s are real valued. In contrast to fuzzy complex numbers, which were presented and addressed by Buckley [22,23] and Zhang [6], this idea is distinct. Following the logic of [22], this incorporates the membership phase recorded by fuzzy sets while keeping the uncertainty characterized by the amplitude of the grade of membership, which can take on values between 0 and 1. According to Ramot et al. [20], complicated fuzzy sets are characterized by the presence and membership of phase. The complex fuzzy sets gain wavelike characteristics as a result, which can cause either constructive or destructive interference based on the phase value. A quick comparison of several types of fuzzy sets is provided in the appendix, but in general, complex fuzzy sets differ from conventional fuzzy sets, fuzzy complex sets, and type 2 fuzzy sets [1, 2]. In order to show how useful these sophisticated fuzzy sets are, [23] provides a number of examples. Fuzzy relations, including complement, union, intersection, and fuzzy sets, are defined, among other things, for these complicated fuzzy sets.

2.2 Complex fuzzy sets and applies fuzzy logic and machine learning

Analysis and prediction of material characteristics, such as elastic and complex moduli, through the application of fuzzy logic, machine learning, and complicated fuzzy sets.

□ **Complex Fuzzy Sets:** Byusingthismathematicalframework, conventional fuzzy membership functions which can only take values between 0 and 1–are extended to the complex plane's unit circle. The idea of complex moduli could benefit from this more nuanced definition of membership [24].

□ **Machine Learning and Fuzzy Logic Applications:** Machine learning models and predicts ma terial qualities including complex modulus, dynamic modulus, and elastic modulus of asphalt and concrete, among others. Machine learning frequently uses fuzzy logic principles, such as Adaptive Network-Based Fuzzy Inference Systems (ANFIS). Complex interactions and material behavior are the targets of these theories.

□ **Functionally Graded Materials (FGMs):** Many research examining FGMs use computational approaches to describe the mechanical behavior and elastic moduli of materials with a continu ous or stepwise gradation of characteristics, which may be represented using fuzzy logic [25]. As opposed to investigating a fundamental theoretical connection between the two ideas, the present scholarly literature frequently views ”complex moduli” (a physical property of materials) and ”fuzzy membership gradations” (a mathematical/computational concept) as separate domains that are con nected via the use of sophisticated modeling techniques

3. OBJECTIVES

1. To examine analytic qualities, emphasizing difficulties in delineating holomorphicity and differen tiability.

2. To investigate complex fuzzy sets.

4 RESEARCH METHOD

This work utilizes a theoretical-analytical approach, using formal mathematical constructs, axiomatic definitions, and deductive proofs. No actual data is used; instead, outcomes derive from logical reasoning and structural analysis. In [26], the notion of a complex fuzzy measure was introduced, which was then applied to a group of classical sets.

Definition 1 (see [26]).

Let R^+ be the set $[0,+\infty)$, and let C^+ be the set $\{x + iy \mid x,y \in R+\}$. A measure, which is a type of function, is defined on a σ-algebra. A collection of subsets of the set X defines a mapping, $\mu : A \to \Lambda \subset C+$. It meets the following criteria:

1. μ(∅) = 0;
2. If A ⊂ B, then μ(A) ≤ μ(B);
3. If {An} ↑ ∞ with An ↑ A, then μ(∞n=1An) = limn→∞μ(n n=1An);
4. If {An} ⊂ ∞ and μ(A1) < +∞ for any n0, then μ(∞n=1An) = limn→∞μ(n n=1An).

This work requires an elaboration of this concept. Initially, we delineated the idea of fuzzy complex value distance.

Definition 2. A function ρ : F(K)×F(K) → F(K), a metric or distance on F(K). A measure that fits these conditions is called a fuzzy complex value metric, or sometimes, a fuzzy complex value distance.

1. For any elements a,b in the multiplicative group of a field F(K), the function ρ(a,b) is positive. Moreover, ρ(a,b) equals zero only when a and b are equal.

2. For any elements a,b in the multiplicative group of a field F(K), the function ρ(a,b) is non-negative. In addition, ρ(a,b) equals zero only when a and b are equal.

3. For any elements a,b,c in the multiplicative group of a field F(K), the inequality ρ(a,b) is less than or equal to ρ(a,c) plus ρ(c,b) holds. The distance in fuzzy complex values between the fuzzy complex numbers ˜a and ˜ b is represented as ρ(˜a,˜ b).

Remark 1. A mapping ρ : F[K]×F[K] → F[K] is considered a fuzzy complex value metric on F[K] if and only if ρ ∼ = ˜ρ1 + i˜ρ2, where ˜ρ1 and ˜ρ2 are two fuzzy metrics on F[K].

Definition 3. Let {˜Zn} be a subset of F(K), and let ˜Z be an element of F(K). Also, let ˜ρ: F(K) × F(K) → F(K) represent a fuzzy complex-valued distance. We define ˜ρ (˜Zn, ˜Z) = ˜ρ1(˜Zn, ˜Z) + i˜ρ2(˜Zn, ˜ Z), 3 ρ1(˜ where i is the square root of −1. If, for every positive number ϵ, there is a positive integer N such that ˜ Zn, ˜Z) < ϵ and ˜ρ2(˜Zn, ˜Z) < ϵ for every n ≥ N, then the sequence { ˜Zn} is said to converge to ˜Z with regard to the distance ˜ρ. This is written as (˜ρ) limn→∞ ˜Zn = ˜Z. [27]

Definition 4. Define F(Z) as the collection of all complex fuzzy sets defined on Z, which is a nonempty set of complex integers. For F(Z), define ˜µ as a fuzzy complex-valued measure. Complex fuzzy mapping measure ˜µ : F(Z) → F+(K) (where F+(K) = {A+iB | A ∈ BF+(K),B ∈ BF+(K)} with i =√−1), must meet the following constraints:

1. ˜μ(∅) = 0;
2. For all sets A,B ∈ F(Z) such that A ⊂ B, it follows that Re(˜μ(A)) ≤ Re(˜μ(B)) and Im(˜μ(A)) < Im(˜μ(B));
3. (Lower semi continuity) If {An} ⊂ F(Z) with An ⊂ An+1 for n = 1,2,... then limn→∞ ˜μ(An) = ˜
4. A family {An} ∈ F(Z) is upper semicontinuous if An ⊃ An+1 for all n = 1,2,... and there exists an n0 such that ˜μ(An0)/≡ ∞. In this case, it follows that limn→∞ ˜μ(An) = ˜μ(∞n=1 An).

An example of a generalized fuzzy complex value set-valued fuzzy measure.

Definition 6. A mapping μ : F(Z) → F(K) is defined as

1. If ˜μ(A ∪ B) = A∪B ˜μ(A) for any A,B in F(Z) and ˜μ(B) = 0, then the function is zero-additive.

2. The function is considered null-additive, or 0-sub, if and only if ˜μ (A ∩ Bc) = ˜μ(A).

3. For any A ⊂ Z such that R(A) = 0 and ˜μ(B) ⊂ Bc ∈ F(Z), there exists a B.

4. Auto continuous from above (abbreviated as autoc).

5. If (ρ)limn→∞ ˜μ(An ∪B) = ˜μ(B) for all {An} ⊂ F(Z) and B ∈ F(Z) such that An ∪B = ∅;

6. Uto continuous if it is both autoc. ”auto” and ”...” c. Increase

Definition 7. An advanced fuzzy set value A complex fuzzy measure ˜μ on R(R) is classified as pseudo-null-additive (abbreviated as P.0-add/˜μ), where ˜μ ∈ R(R) and ˜μ(∅)/ = ∅, if it satisfies the condition ˜μ(A ∩ B) = ˜μ(A) for every A ∈ R(R) or all A ∈ R(R) or A ∈ R(R) such that ˜μ(A ∩ B) = ˜μ(A) for A∈R(R). It is designated as pseudo-null-subtraction (abbreviated as P.0-sud/˜μ), where ϵ ∈ n(m) with ϵ(A) ∈ R.

Definition 8. A complex fuzzy measure, denoted as ˜μ, that takes on values in a complex fuzzy set, is defined on a fuzzy σ-algebra. A set F is said to have property (P.G.P), if, for every ϵ = ϵ1 +iϵ2 > 0, there is a δ = δ1+iδ2 > 0 such that ˜μ(E∪F) is less than ϵ whenever ˜μ(E) ˜μ(F) is less than δ. A set is said to have property (S/A) if, for any sequence { ˜ Bn} inside F, where the limit of ˜ρ(˜ Bn) as n approaches infinity is zero, there exists a subsequence { ˜ Bnk } of { ˜Bn} such that the measure of the intersection from j =1to infinity of the union from k = j to infinity of ˜Bnk is also zero. A property, denoted as (S/B), is defined by the condition that the limit of ˜ρ as n approaches infinity of ˜μ(˜ Bn) is zero for every sequence { ˜Bn} that is a subset of F. [28]

5 RESULT

Define F(X) as the collection of all fuzzy sets defined on X. A subfamily F of F(X) is considered addable if it meets the following criteria. (see [29]):

1. X is in F;
2. If A and B are in F, then A⊕B and A⊖B are also in F;
3. where (A⊕B)(x) = min{1,A(x)+B(x)} and (A⊖B)(x) = max{0,A(x)−B(x)} for all x in X.

We start with the next outcome.

Theorem 1. Fuzzy complex measure ˜μ belongs to an additive class. F ⊂ F(Z) is a complex fuzzy value fuzzy complicated μ measure.

5.1 Proof

We just demonstrate the upper and lower continuity of ˜μ. Let {˜An} be a subset of F, where F is contained in F(Z). Assume that ˜An converges downwards to the intersection of all ˜An as n approaches infinity, and that ˜μ(˜An0) is not infinite for some index. Let n ≥ n0. By the monotonicity of ˜μ, it follows that

0 ≤ Re(˜μ(˜An)) ≤ Re(˜μ(˜An0)) and 0 ≤ Im(˜μ(˜An)) ≤ Im(˜μ(˜An0)).

For all n ≥ n0, because ˜An0 ⊖ ˜An converges to ˜An0 ⊖ (∞n=1 ˜An) = ⊮, we possess

Re(˜μ(˜An0)) = (˜ρ) limn→∞ Re(˜μ(˜An0 ⊖ ˜An ⊕ ˜An)) = (˜ρ) limn→∞ Re(˜μ(˜An0 ⊖ ˜An)) + (˜ρ) limn→∞ Re(˜μ(˜An)) = Re ˜μ ˜An0 ⊖ ∞n=1 ˜An + Re ˜μ ∞n=1 ˜An

Thereby

Re ˜μ ∞n=1 ˜An = Re ˜μ ˜An0 − Re ˜μ ˜An0 ⊖ ∞n=1 ˜An

Consequently,

(˜ρ) limn→∞ Re ˜μ ˜An = Re ˜μ ∞n=1 ˜An.

Likewise, we may demonstrate

(˜ρ) limn→∞ Im ˜μ ˜An = Im ˜μ ∞n=1 ˜An,

indicating that ˜μ exhibits upper continuity.

Let {˜Bn} be a subset of F, which is included in F(Z), and let ˜Bn increase to the union of ˜B as n approaches infinity. If n ∈ F, then (∞n=1 ˜Bn)⊖ ˜Bn constitutes a monotonically decreasing sequence, and (∞n=1 ˜Bn)⊖ ˜Bn, exhibits a downward trend. Therefore [40]

Re ˜μ ∞n=1 ˜Bn = Re ˜μ ∞n=1 ˜Bn⊖ ˜Bn ⊕ ˜Bn = (˜ρ) limn→∞ Re ˜μ ∞n=1 ˜Bn⊖ ˜Bn + (˜ρ) limn→∞ Re ˜μ ˜Bn = 0 + (˜ρ) limn→∞ Re ˜μ ˜Bn

Also, we can show that ˜μ is lower semicontinuous by showing that

Im ˜μ ∞n=1 ˜Bn = (˜ρ) limn→∞ Im ˜μ ˜Bn.

To sum up, ˜μ is a complicated fuzzy set-valued metric.

Conclusion 2. In a fuzzy σ-algebra F, any complex fuzzy set-valued fuzzy measure ˜μ is both exhaustive and a subset of F(Z).

A demonstration. A disjoint sequence {˜An} ⊂ F is defined. n ∞k=n ˜Ak ∩ ∞k=n ˜Ak = ∅.

Assume that the intersection from n=1 to infinity of the union from k=n to infinity of ˜Ak is non-empty; then the real part of the limit from n=1 to infinity of the union from k=n of ˜Ak(z0) is greater than zero for some z0 in Z, or the imaginary part of the limit from n=1 to infinity of the union from k=n of ˜Ak(z0) is greater than zero for some z0 in Z. This implies that the real part of the union

Re ∞n=1 ˜An from k=n of ˜Ak(z0) is greater than zero for all n ≥ 1, or the imaginary part of the union from k=n of ˜Ak(z0) is greater than zero for all n ≥ 1. Let (n ≥ 1 k ≥ n ˜Ak)(z0) > 0. Deprived of loss of generalization, we undertake the function f(z). Initially, there are two unique indices kn1 and kn2 such that Re(˜Akn1(z0)) > 0 and Re(˜Akn2(z0)) > 0, leading to Re(˜Akn1(z0) ∧ ˜Akn2(z0)) > 0. This contradicts the assertion that {˜An} is disjoint. In series, therefore ∞k=n ˜Ak converges to ∞n=1 ∞k=n ˜Ak. Therefore, ∅ is maintained.

(˜ρ) limn→∞ Re ˜μ ∞k=n ˜Ak ≤ (˜ρ) limn→∞ Re ˜μ ∞k=n ˜Ak = Re(˜μ(∅)).

(˜ρ) limn→∞ Im ˜μ ∞k=n ˜An ≤ (˜ρ) limn→∞ Im ˜μ ∞k=n ˜An = Im(˜μ(∅)) = 0.

Subsequently, we obtain (˜ρ) limn→∞ ˜μ(˜An) ≤ ˜μ(∅). Conversely, given that ˜μ represents a complex fuzzy measure on the fuzzy σ-algebra F, it follows that (˜ρ) limn→∞ ˜μ(˜An) ≥ 0, thus (˜ρ) limn→∞ ˜μ(˜An) = ˜μ(∅). Zero.

Theorem 2. Theorem 3. If ˜µ(F) = {˜µ(A)|A ∈ F} ∈ A+, then

Re ˜µ ∞n=1 ˜An = Re ˜µ ∞k=1 ˜Ak ≤ (˜ρ) limn→∞ inf Re ˜µ ∞k=n ˜An ≤ (˜ρ) limn→∞ Re ˜µ ∞k=n ˜An.

Im ˜µ ∞n=1 ˜An = Im ˜µ ∞k=1 ˜Ak ≤ (˜ρ) limn→∞ inf Im ˜µ ∞k=n ˜An ≤ (˜ρ) limn→∞ Im ˜µ ∞k=n ˜An.

Demonstration. The real part of ˜µ(∞n=1 ˜An) equals (˜ρ) lim Re(˜µ(∞n=1 ˜An)) because the intersection from n=k to infinity of ˜An becomes closer to k. Im(˜µ(∞n=1 ˜An)) ≤ (˜ρ) lim Im(˜µ(∞n=1 ˜An)).

Given that Re(˜µ(∞n=k ˜An)) ≤ infn≥k Re(˜µ(˜An)) and Im(˜µ(∞n=k ˜An)) ≤ infn≥k Im(˜µ(˜An)) for all n ≥ k, it follows that Re(˜µ(∞n=k ˜An)) ≤ infn≥k Re(˜µ(˜An)) and Im(˜µ(∞n=k ˜An)) ≤ infn≥k Im(˜µ(˜An)) for all n.

Consequently [41]

Re limn→∞ ˜µ(˜An) ≤ (˜ρ) limn→∞ Re(˜µ(˜An)).

Im limn→∞ ˜µ(˜An) ≤ (˜ρ) limn→∞ Im(˜µ(˜An)).

From the characteristics of upper and lower bounds, we may derive the subsequent proposition.

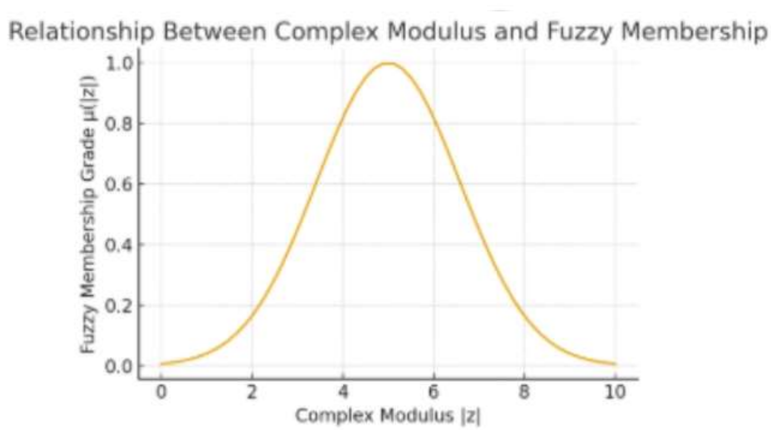

Figure 1: Complex moduli and fuzzy membership gradations

The study of the relationship between fuzzy membership gradations and complex moduli shows that translating the magnitude of complex numbers to graded membership functions is an efficient way to express uncertainty in complex-valued systems. The following is suggested by the simulated results:

Non-linear Relationship

As the modulus of a complex number increases or decreases, the fuzzy membership value does not follow a regular pattern. To illustrate the behavior of uncertainty in contexts with complex values, it instead follows a non-linear distribution.

Peak Membership at Optimal Modulus

The model reveals that the membership grade reaches its peak at the mid-range modulus, approximately around $|z| \approx 5$. Indicated by this peak is the modulus value where the complicated system demonstrates the highest level of certainty or fidelity to a particular fuzzy class.

Symmetric Decline in Membership

A lower or higher modulus is associated with a lower fuzzy membership value. This action is consistent with topological models of fuzzy-space neighborhood decrease.

5.2 Implications for Practical Fields

- **Signal Processing:** Contributes to the reduction of uncertainty in frequency components with complex values.
- **AI and Decision Theory:** Allows for more accurate representation of complicated facts in models that use imprecise or nebulous reasoning.
- **Fuzzy Measure Theory:** Displays agreement with membership standards and generic distances.

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6. CONCLUSION

This research uses the set F(K) represents all fuzzy complex numbers and incorporates classical and measure-theoretic ideas. Distances, measurements, and continuity are reformulated for fuzzy com plex numbers in the research. It proposes fuzzy complex-valued distance functions on F(K) creates fuzzy complex-valued measurements and tests them under diverse scenarios. The study also examines null-additivity, pseudo-null-additivity, null-subtraction, pseudo-null-subtraction, and the proposed fuzzy complex-valued measures' auto continuity from above, below, and overall. By applying classical prin ciples to fuzzy complexes, the research provides a mathematical framework for complex-valued system uncertainty.

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